



LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Eight: Functions and Relations

Lesson

15

Working with Functions and Relations

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 7i page 312)

INTRODUCTION

The ability to work with functions and relations will assist the students in making sense of a number of mathematical manipulations. In this lesson we will also distinguish between a function and a relation.

OBJECTIVES

At the end of this lesson you will be able to:

- a) Define a relation
- b) Use arrow diagrams to show relations
- c) Define a function
- d) Distinguish between the graph of a relation and the graph of a function





8.1 Definition of a relation, function, mapping

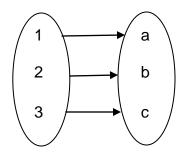
Relations

A relation is a set of ordered pairs. For example: (1, a), (2, b), (3, c). The set of first elements is called the domain: {1, 2, 3} and the set of second elements is called the range: {a, b, c}.

Arrow or Mapping Diagrams

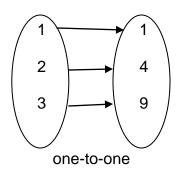
Relations can be represented on arrow diagrams.

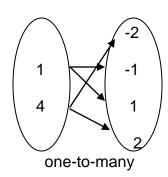
Example:

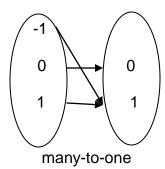


Types of relations

The three main types of relations are shown in the following arrow diagrams.





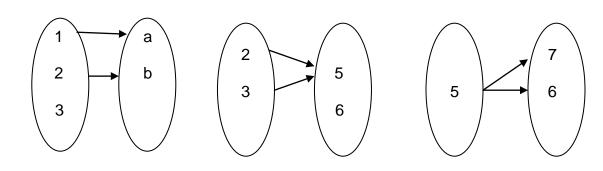


Functions

A function is a one-to-one or many-to-one relation. Therefore the first and third arrow diagrams directly above represent functions. Each element in the domain must be mapped onto one and only one element in the range. The diagram in the middle is not a function because an element in the domain cannot be mapped on to two elements in the range.



1. Which of the following arrow diagrams represent a function?

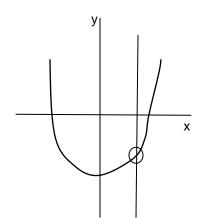


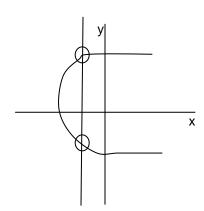
2. Represent the following relation on an arrow diagram:

$$(0,0)$$
, $(1,3)$, $(2,3)$, $(3,81)$, $(1,4)$.

8.2 Distinguishing between a function and a relation

You can distinguish between a function and a relation from their graphs by drawing a vertical line anywhere on the graph. If the line touches the graph at more than one point then it is not a function. However the graph is still a relation.

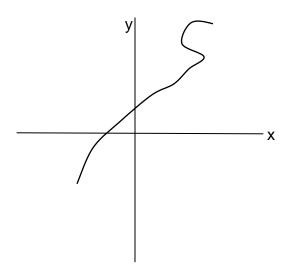




The first graph represents a function. Anywhere you draw a vertical line it will only touch the graph once. The second graph is not a function. The vertical line touches the graph at two points. This means one element in the domain (x-axis) has two images in the range (y-axis).



Determine if the following graph represents a function.



8.3 Forms of Functions and Relations

Functions can be linear, quadratic, rational etc.

Linear Functions

Example: $f:x \to 5x + 2$ or f(x) = 5x + 2.

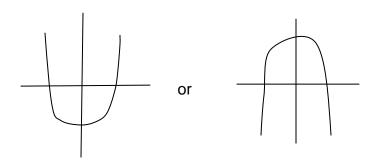
This means that some value $^{\mathcal{X}}$ in the domain is mapped onto $\mathcal{Y}=\mathbf{5}\mathcal{X}+\mathbf{2}$, where \mathcal{Y} is an element in the range. The graph of a linear function is a straight line.

Quadratic Functions

The highest power of X is 2.

Example:
$$f(x) = 3x^2 + 2x - 1$$
.

The graph of a quadratic function is a parabola:



Rational Functions

Example:
$$f(x) = \frac{2x+1}{x-1}$$

These functions are not defined for any value of x that makes the denominator zero. Therefore the above function has no value when x = 1.

Evaluating Functions

The functions can be evaluated by substituting values for $^{\mathcal{X}}$ and working out what $^{\mathcal{Y}}$ will be using the rule.

Example: Given $f: X \to 5X + 2$, what number in the image set is 2 mapped to.

Solution:

$$f(2) = 5(2) + 2$$

= 10 + 2
= 12

Therefore $2 \rightarrow 12$.



The function f is defined by $f:x \to x^2 + 5$, where $x \in \{1,2,3,4\}$.

Do you agree that f(3) = 14?

State the value of each of the following:

(a) f(1) (b) f(2) (c) f(4) (Ex 7i page 312)



CXC Question

Given that

$$g: \rightarrow \frac{x+2}{x-5}$$

- (a) Calculate g(2).
- (b) State the value of 1 for which g(x) is not defined.

Conclusion

Functions and relations where defined in this lesson. We also looked at evaluating functions. In the lesson that follows we will look at composition of functions.