To **solve an inequality in one variable,** first change it to an equation and solve. Place the solution, called a "boundary point," on a number line. This point separates the number line into two regions. The boundary point is included in ($\geq$ or $\leq$ shown by a solid dot) or excluded from (> or < shown by an open dot) the solution depending on the inequality sign. Next, choose a number from each region and check if it is true or false in the original inequality. If it is true, then every number in that region is a solution to the inequality. If it is false, then no number in that region is a solution to the inequality. See the Math Notes box on page 386.

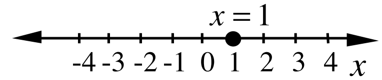
Example 1

Solve $3x-(x+2)\geq 0$

Change to an equation and solve.

$$3x-(x+2)=0$$
$$3x-x-2=0$$
$$2x=2$$
$$x=1$$

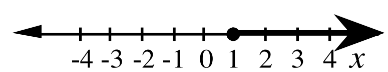
Place the solution (boundary point) on the number line. Because $x=1$ is also a solution to the inequality ($\geq$), we use a solid dot.



Test a number from each side of the boundary point in the original inequality.

Test $x=0$	Test $x=3$
$3\cdot 0-(0+2)\geq 0$	$3\cdot 3-(3+2)\geq 0$
$-2\geq 0$	$4\geq 0$
False	True

The solution is: $x\geq 1$.



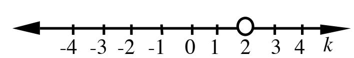
Example 2

Solve: $-x+6>x+2$

Change to an equation and solve.

$$-x+6=x+2$$
$$-2x=-4$$
$$x=2$$

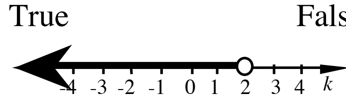
Place the solution (boundary point) on the number line. Because the original problem is a strict inequality (>), $x=2$ is not a solution, so we use an open dot.



Test a number from each side of the boundary point in the original inequality.

Test $x=0$	Test $x=4$
$-0+6>0+2$	$-4+6>4+2$
$6>2$	$2>6$
True	False

The solution is: $x<2$.



Problems

Solve each inequality.

1.	$4x-1\geq 7$	2.	$2(x-5)\leq 8$	3.	$3-2x<x+6$
4.	$\frac{1}{2}x>5$	5.	$3(x+4)>12$	6.	$2x-7\leq 5-4x$
7.	$3x+2<11$	8.	$4(x-6)\geq 20$	9.	$\frac{1}{4}x<2$
10.	$12-3x>2x+1$	11.	$\frac{x-5}{7}\leq -3$	12.	$3(5-x)\geq 7x-1$
13.	$3y-(2y+2)\leq 7$	14.	$\frac{m+2}{5}<\frac{2m}{3}$	15.	$\frac{m-2}{3}\geq\frac{2m+1}{7}$

Answers

1.	$x\geq 2$	2.	$x\leq 9$	3.	$x>-1$
4.	$x>10$	5.	$x>0$	6.	$x\leq 2$
7.	$x<3$	8.	$x\geq 11$	9.	$x<8$
10.	$x<\frac{11}{5}$	11.	$x\leq -16$	12.	$x\leq 1.6$
13.	$y\leq 9$	14.	$m>\frac{6}{7}$	15.	$m\geq 17$

GRAPHING INEQUALITIES 9.2.1 and 9.2.2

To **graph an inequality** in two variables, first graph the corresponding equation. This graph is know as the boundary line (or curve), since all points that make the inequality true lie on one side or the other of the line. Before you graph the equation, decide whether the line or curve is part of the solution or not, that is, whether it is solid or dashed. If the inequality symbol is either ≤ or ≥, then the boundary line is part of the inequality and it must be solid. If the symbol is either < or >, then the boundary line is dashed.

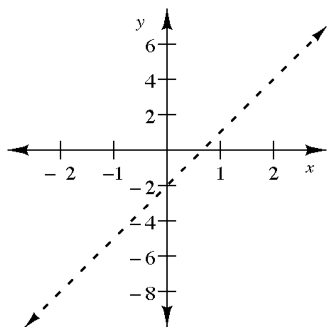
Next, decide which side of the boundary line must be shaded to show the part of the graph that represent all values that make the inequality true. Choose a point not on the boundary line. Substitute this point into the **origina**l inequality. If the inequality is true for the test point, then shade the graph on this side of the boundary line. If the inequality is false for the test point, then shade the opposite side of the line. See the Math Notes box on page 393.

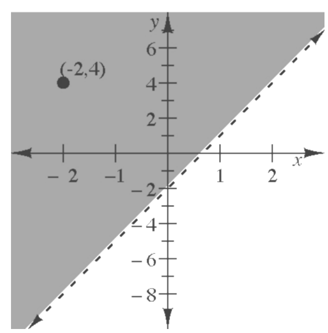
CAUTION: If you need to rearrange the inequality in order to graph it, such as putting it in slope-intercept form, always use the **original** inequality to test a point, NOT the rearranged form.

Example 1

Graph the inequality $y > 3x - 2$.
First, graph the line $y = 3x - 2$, but draw it dashed since > means the boundary line is not part of the solution.

Next, test the point (-2, 4) that lies to the left of the boundary line.





$4 > 3(-2) - 2$, so $4 > -8$.
Since the inequality is true for this test point, shade the region left of the boundary line.

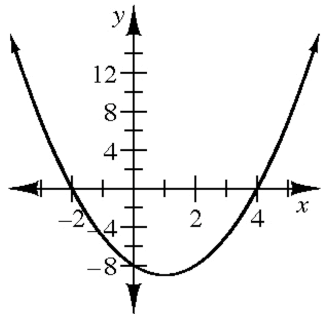
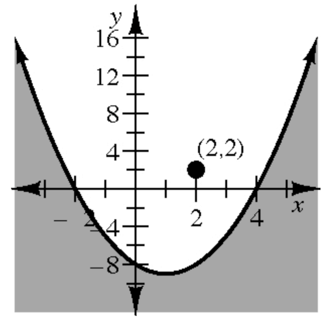
Example 2

Graph the inequality $y \le x^2 - 2x - 8$.

First, graph the parabola $y = x^2 - 2x - 8$ and draw it solid, since ≤ means the boundary curve is part of the solution.

Next, test the point (2, 2) above the boundary curve.

$2 \le 2^2 - 2 \cdot 2 - 8$, so $2 \le -8$

Since the inequality is false for this test point above the curve, shade below the boundary curve.

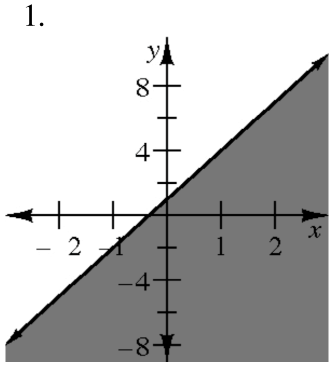
Problems

Graph each of the following inequalities on a separate set of axes.

1.	$y \le 3x + 1$	2.	$y \ge -2x + 3$	3.	$y > 4x - 2$
4.	$y < -3x - 5$	5.	$y \le 3$	6.	$x > 1$
7.	$y > \frac{2}{3}x + 8$	8.	$y < -\frac{3}{5}x - 7$	9.	$3x + 2y \ge 7$
10.	$-4x + 2y < 3$	11.	$y \ge x^2 - 3$	12.	$y \le x^2 + 2x$
13.	$y < 4 - x^2$	14.	$y \le \lvert x + 2 \rvert$	15.	$y \ge -\lvert x \rvert + 3$

Answers

1.



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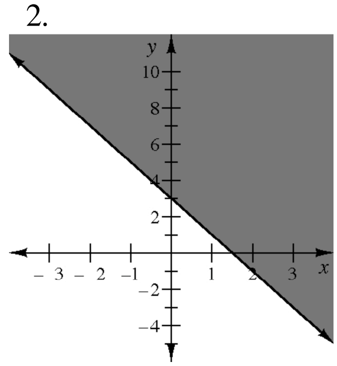

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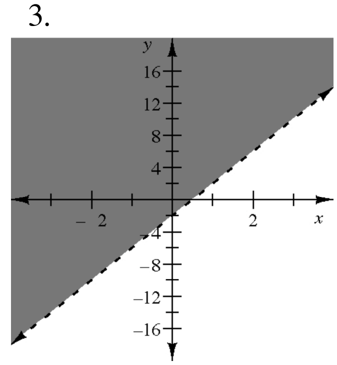

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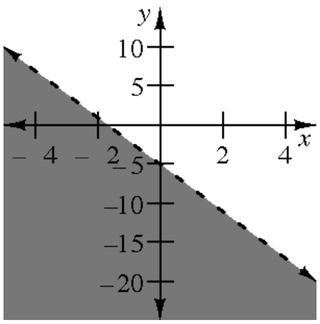

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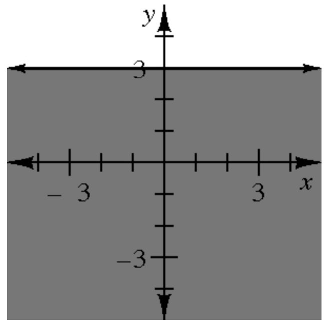

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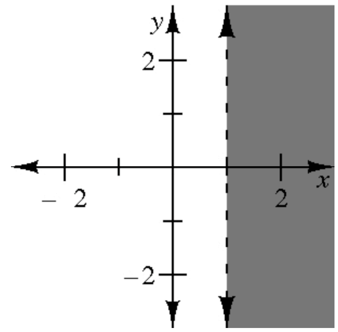

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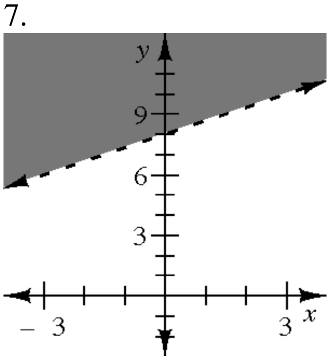

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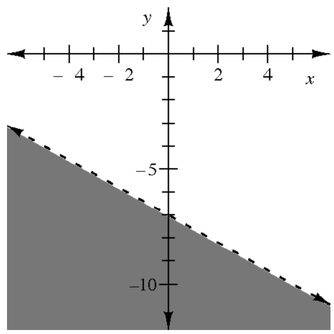

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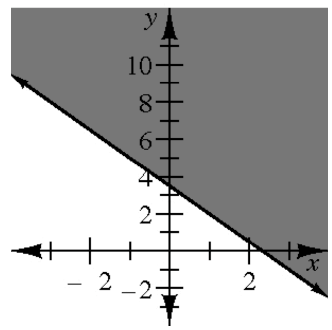

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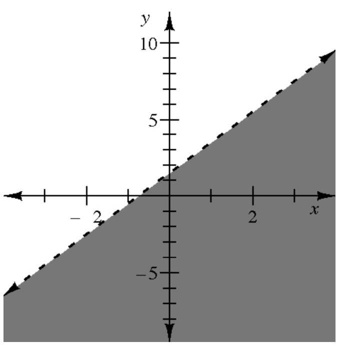

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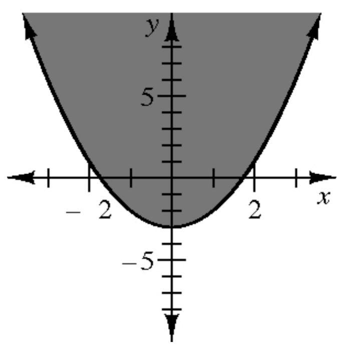

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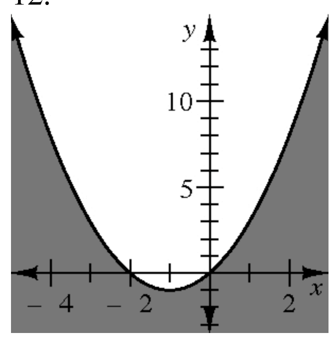

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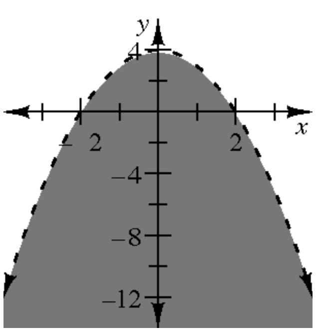

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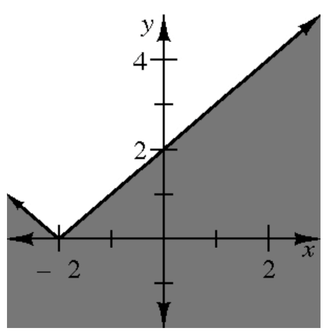

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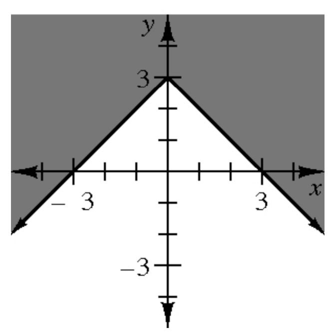
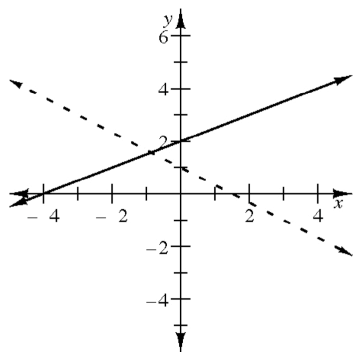


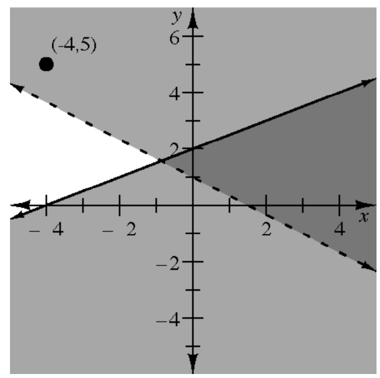
To **graph systems of inequalities**, follow the same procedure outlined in the previous section but do it twice—once for each inequality. The solution to the system of inequalities is the **overlap** of the shading from the individual inequalities.

Example 1

Graph the system $y \le \frac{1}{2}x + 2$ and $y > -\frac{2}{3}x + 1$.

Graph the lines $y = \frac{1}{2}x + 2$ and $y = -\frac{2}{3}x + 1$. The first is solid and the second is dashed. Test the point (-4, 5) in the first inequality.





$5 \le \frac{1}{2}(-4) + 2$, so $5 \le 0$

This inequality is false, so shade on the opposite side of the boundary line from (-4, 5), that is, below the line.

$5 > -\frac{2}{3}(-4) + 1$, so $5 > \frac{11}{3}$

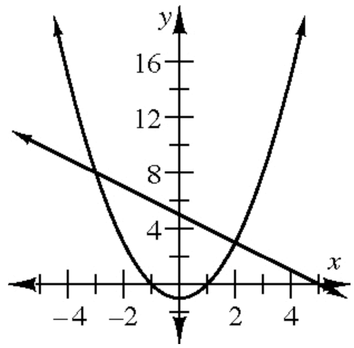
Test the same point in the second inequality. This inequality is true, so shade on the same side of the boundary line as (-4, 5), that is, above the line.

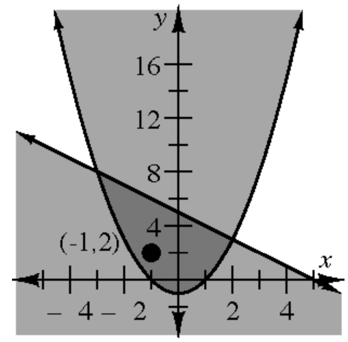
The solution is the overlap of the two shaded regions shown by the darkest shading in the second graph above right.

Example 2

Graph the system $y \le -x+5$ and $y \ge x^2 - 1$.

Graph the line $y = -x + 5$ and the parabola $y = x^2 - 1$ with a solid line and curve.





$2 \le -(-1)+5$, so $2 \le 6$

Test the point (-1, 2) in the first inequality. This inequality is true, so shade on the same side of the boundary line as (-1, 2), that is, below the line.

$2 \ge (-1)^2 - 1$, so $2 \ge 0$

Test the same point in the second inequality. This inequality is also true, so shade on the same side of the boundary curve as (-1, 2), that is, inside the curve.

The solution is the overlap of the two shaded regions shown by the darkest shading in the second graph above right.

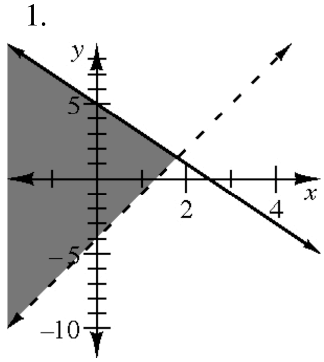
Problems

Graph each of the following pairs of inequalities on the same set of axes.

1. $y > 3x - 4$ *and* $y \le -2x + 5$
2. $y \ge -3x - 6$ *and* $y > 4x - 4$
3. $y < -\frac{3}{5}x + 4$ *and* $y < \frac{1}{3}x + 3$
4. $y < -\frac{3}{7}x - 1$ *and* $y > \frac{4}{5}x + 1$
5. $y < 3$ *and* $y > \frac{1}{2}x + 2$
6. $x \le 3$ *and* $y < \frac{3}{4}x - 4$
7. $y \le 2x + 1$ *and* $y \ge x^2 - 4$
8. $y < -x + 5$ *and* $y \ge x^2 + 1$
9. $y < -x + 6$ *and* $y \ge |x - 2|$
10. $y < -x^2 + 5$ *and* $y \ge |x| - 1$

Answers

1.



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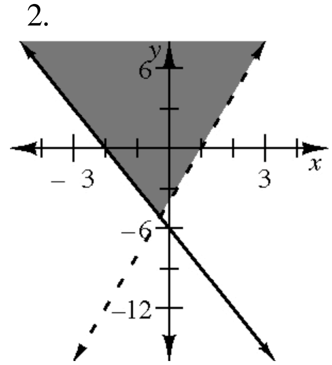

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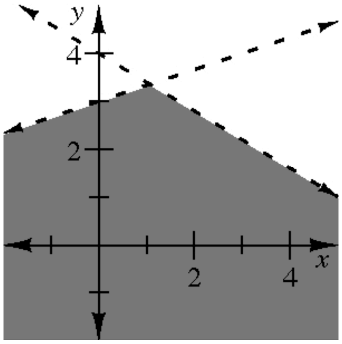
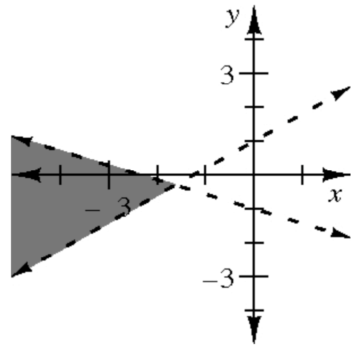
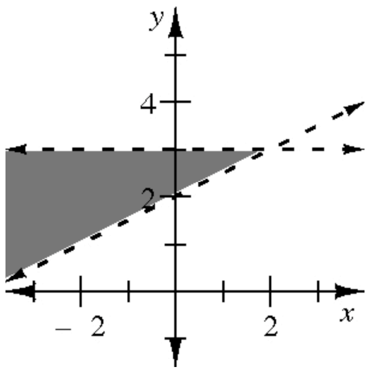

y
4
2
2
4
x

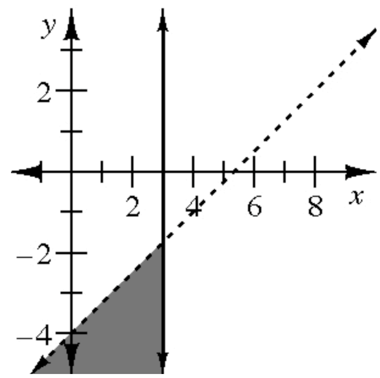
4.


y
3
x
– 3
–3

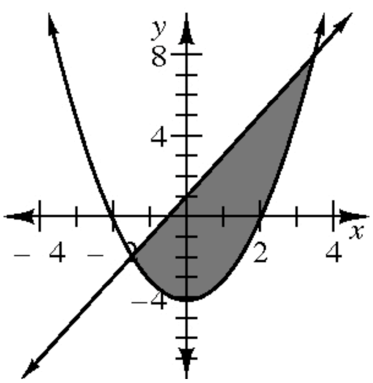
5.


y
4
2
– 2
2
x

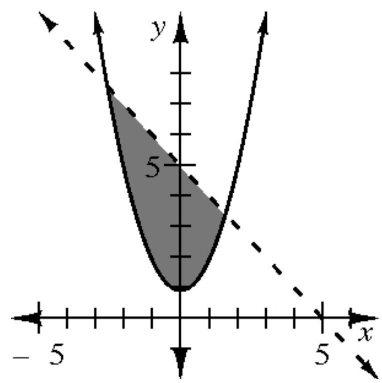
6.


y
2
2
4
6
8
x
–2
–4

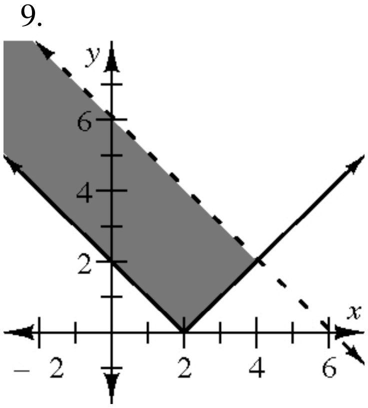
7.


y
8
4
– 4
2
4
x
–4

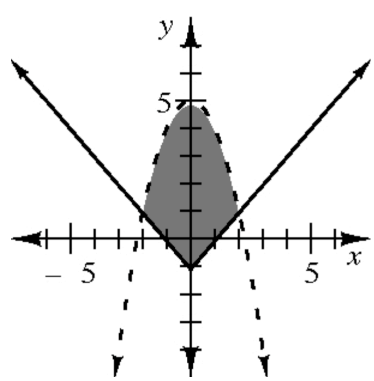
8.


y
5
– 5
5
x

9.


y
6
4
2
– 2
2
4
6
x

10.


y
5
– 5
5
x