KTH Electrical Engineering

Exercises on reliability assessment of electric power systems

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Preface

This compendium is collection of exercises for the course on reliability assessment of electric power systems. The course has been developed within the RCAM group at KTH School of Electrical Engineering.

The first version of the compendium was prepared in 2005, with examples made by master theses students Carl Johan Wallnerström and Otto Wilhelmsson, in co-operation with Lina Bertling. This updated version has been translated to English by Ph.D. student Andrea Lang.

Lina Bertling
Stockholm November 2007.

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Definitions

8760 = Approximate number of hours per year (24*365)
LPi = Load point number i of the analyzed system
$\mathbf{N_{LPi}}$ = Number of customers at LPi
$\boldsymbol{\lambda_{LPi}}$ = Total average failure frequency [failures/year] at LPi
$\mathbf{U_{LPi}}$ = Total average down time [h/year] at LPi
$\mathbf{LOE_{LPi}}$ = Total average undelivered energy at LPi [kWh/year]
$\mathbf{r_{LPi}}$ =Average time for fixing an error in order not to affect LPi

ASAI [probability 0 to 1] = A measure of availability; the number of subscribed hours of delivered energy divided by the number of subscribed hours of wanted energy.

$$ASAI = \frac{\sum_i (N_{LPi}) * 8760 - \sum_i (N_{LPi} * U_{LPi})}{\sum_i (N_{LPi}) * 8760}$$

ASUI [probability 0 to 1] = A measure of unavailability.

$$ASUI = 1 - ASAI$$

SAIFI [failures/year and customer] = Average number of interruptions per year affecting each customer.

$$SAIFI = \frac{\sum_i (N_{LPi} * \lambda_{LPi})}{\sum_i (N_{LPi})}$$

SAIDI [h/year and customer] = Average number of hours per year without electricity for each customer.

$$SAIDI = \frac{\sum_i (N_{LPi} * U_{LPi})}{\sum_i (N_{LPi})}$$

CAIDI [h/failure] = Average length of interruptions.

$$CAIDI = \frac{\sum_i (N_{LPi} * U_{LPi})}{\sum_i (N_{LPi} * \lambda_{LPi})} = \frac{SAIDI}{SAIFI}$$

AENS [kWh/year and customer] = Average annual energy loss for each customer, due to interruptions.

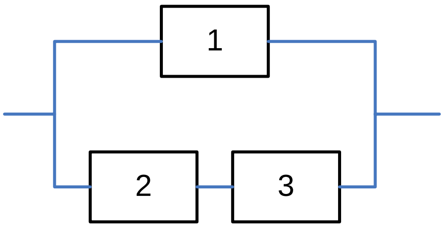
$$AENS = \frac{\sum_i (LOE_{LPi})}{\sum_i (N_{LPi})}$$

1 Reliability calculations for power networks

Problem 1.1
Introduction to reliability calculations for power networks

a) Explain the difference between primary and secondary failures in a power system.
b) Explain the difference between first and second order failures in a power system.
c) Explain the difference between system and component redundancy, and give another example of how redundancy can be created in a power system.
d) Give one example each of active and passive redundancy in a power system, and list some disadvantages with active redundancy.
e) If SAIDI is given as x h/year, and all customers have exactly the same annual consumption, y kWh/year and customer, what is then the AENS?
f) CAIDI is given as x h/failure, and SAIDI is y h/year and customer. What is SAIFI?
g) If ASAI is 0.9999, and all failures always last exactly 1 hour, and affect the whole system, what are SAIDI, SAIFI and ASUI?
h) Why does sometimes CAIDI increase when a redundancy eliminating some of the failures is introduced?
i) What is meant by”the critical state of a component”?

Problem 1.2
Calculations with structure function



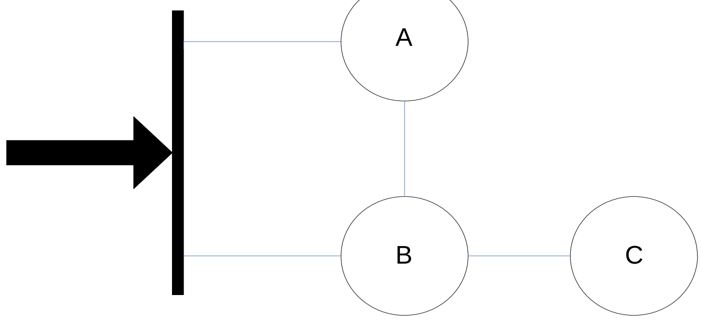
The function probabilities of the components are: p1=0.999, p2=0.998 and p3=0.997

a) Which are the minimal paths and minimal cuts of the system?
b) Calculate the structure function of the system using pivotal decomposition with respect to component 1. What is the function probability of the system?

Problem 1.3
Prioritization of components in a power system
Consider the same system as in Problem 1.2.

a) Rank the components according to structural significance.
b) Rank the components according to Birnbaum’s measure of structural importance $I^{\beta}(i{:}t)$.
c) Rank the components according to critical significance $I^{CR}(i)$.
d) Rank the components according to Vesely-Fussel’s measure of structural importance $I^{VF}(i)$.

Problem 1.4
Calculations of reliability index



Suppose that the figure above symbolizes a larger power system. The black arrow indicates feeding from the grid, where possible errors are neglected. Between the feeding and the areas A and B, there are automatic circuit breakers, which are always in working order. There are three areas with customers which are always connected to each other: A, where 40 % of the customers are situated, B, where 30 % are situated and C, where 30 % are situated. The lines between the areas show how they are connected. Between A and B, there is a disconnecting switch, which it takes 30 minutes to open if a failure occurs. Between B and C there are automatic circuit breakers which with probability 90 % immediately disconnect when a failure occurs. Each area is affected by failure in average once per year and the average outage time is 2 hours. Calculate ASAI, ASUI, SAIFI and SAIDI for the system.

Problem 1.5
Approximate reliability methods
In many reliability computations, different approximations are used to simplify the calculations. In this exercise, you shall find out how large approximations reasonably could be made in a few different situations. An example of a common approximation is the neglecting of those terms in a sum, which are much smaller than the other terms. (Small numbers occur for example when calculating cuts, since they are products of probabilities.)

a) Consider a radial circuit modeled by three components connected in series. If one component fails, this whole subsystem fails. The probability for a failure of the subsystem is the sum of all probabilities of failure for the considered components, minus the mathematical cuts: P1 + P2 + P3 - P1*P2 - P1*P3 - P2*P3 - P1*P2*P3, where P1 = 0.001, P2 = 0.002 and P3 = 0.003. How large will the overestimation and the underestimation respectively be, if no consideration is taken first to the cut of three components, and then to all mathematical cuts?

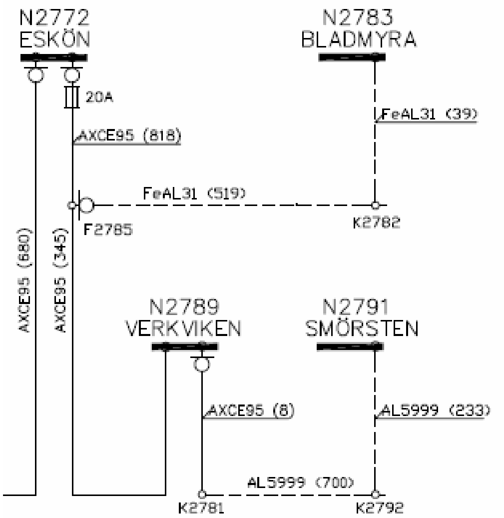
b) RADPOW: Make approximations, e.g. when calculating the failure rate for failures of the second order: $\lambda_{xy} = \frac{(\lambda_x * \lambda_y) * (r_x + r_y)}{1 + \lambda_x * r_x + \lambda_y * r_y} \approx (\lambda_x * \lambda_y) * (r_x + r_y)$. Suppose that only approximations causing a deviation of at most 0.1 % are accepted. Suppose that r=2/8760 years for both components, and that the failure rate for x is twice as big as the one for y. For how large values of the failure rate can the approximate equation be accepted?

c) Redo exercise b), but now trying deviation limits of 1 % and 0.01 %.

d) Redo exercise b), now supposing first that the failure rates for x and y are the same, and then that the failure rate for x is 20 times the one for y. Draw a general conclusion concerning the reliability of the approximate method, based on the results from exercises b) , c) and d).

Problem 1.6

Modeling and computation of reliability for a smaller part of a real power system



The figure illustrates a small part of a real distribution area at the 10 kV-level. You shall draw up a suitable model of the grid, and then make a few reliability calculations. In order to simplify things, it is supposed that Eskön is connected at a transformer point to a ”strong” grid, where the feeding never fails (through the 680 m cable that in the figure ends up in nowhere.)

Fundamental conditions:

- Dashed lines represent overhead lines, with a mean failure frequency of 0.20 failures/year and km, and a mean outage time of 2.5 hours.
- Solid lines represent underground cables with a mean failure frequency of 0.02 failures/year and km, and a mean outage time of 2.5 hours.
- For both lines and cables, the number in brackets indicates the length in meters.
- In case of a failure in a line or a cable, the fuse (20A) will be released with probability 40 %. The main purpose of the fuse is to fulfill the releasing conditions for the grid - not to stop failures. It can, though, isolate failures immediately.
- The disconnecting switches (round open circles) are normally closed but can be opened after on average 1 hour, and are approximately assumed never to fail.
- The medium voltage transformer stations (thick short line marked by the number N27XX and a name) have a failure frequency of 0.05 failures/year and an average outage time of 3 hours. Failures in the transformer station affect, unlike other failures, only underlying customers.
- Remaining parts of the system are considered as ideal.

N2772: 51 customers, 349 532 kWh/year (values are authentic)
N2783: 33 customers, 183 009 kWh/ year
N2789: 10 customers, 142 837 kWh/ year
N2791: 13 customers, 74 220 kWh/ year

a) How is SAIFI affected if the disconnecting switches are removed?
b) Make a network model of the system, based on the figure and given information.
c) Calculate SAIDI, SAIFI, CAIDI and AENS for the primary case.
d) Calculate how these reliability indices would change, if all overhead lines were replaced by underground cables.
e) Consider again the primary case, but with the difference that the disconnecting switches now are perfect, and therefore always disconnect the failures.
f) Suppose that the subsystem is built so that it can be fed also from another place, and more precisely to K2792. Suppose further that the disconnecting switch at the medium voltage transformer station Verkviken normally is open, and that no further sources of failure will arise, thanks to the possibility to feed the system from two directions.
g) Based on the results from c)-f), discuss different redundancy alternatives for the system.

Problem 1.7

Calculation of overload aspects in a smaller electrical grid

A large industry gets its electrical energy through two parallel lines, which earlier implied redundancy. Recently, the industry doubled its production, and has since then an increased power demand. Today, one line alone is not sufficient to feed of the industry; a failure in one line causes overload and a complete stop for the whole industry. You have been given the task to look at two different investment alternatives, in order to solve this problem:

1: The more economical alternative would be to install a third line of the same sort as the other two, and parallel to those, which would result in a so called 2/3 system. Such a system is in working order as long as at least two of the three lines are in working order.
2: The second and more expensive alternative is to install a more powerful line parallel to the other two; a line that alone can bear the entire load. As long as the new line is in order, this system is working, and if not, it still works if both of the other lines are working at the same time.

a) Find minimal paths and minimal cuts for the two alternatives.
b) Make two, concerning reliability equivalent, network models – one for each system. Part from the results in a).
c) Find the structure function for each alternative. (In fundamental form.)
d) Suppose that all lines have probability p to be in operation. For each alternative, write the probability, expressed in p, for the system to be in operation.
e) Suppose that p=0.99 and that every hour of down time costs the company 100 000 SEK. How much lower must the average annual cost (including all costs, such as loans, write-off costs etc.) for the first alternative be, in order for this to be profitable?
f) Now suppose that failures occur with a yet smaller probability, but that they on the other hand are more expensive. The industry would like to have an estimation of how long it will take until the first failure occurs (mean time to failure) for the two alternatives. Each line is supposed to have a failure frequency of 0.02 failures/year. How many more years, expressed in percent, should it in average take before a failure occurs in the more expensive system, compared to the more economic one? (Broken lines are not repaired.)

Problem 1.8

Comparison of maintenance strategies

An important node in a power distribution network is fed through two parallel cables. The cables are so badly placed, that every time a repair work has to be done, there is a cost of 100 000 SEK for each cable that has to be repaired. Furthermore, if only one cable is broken, there is always a risk that the unbroken cable breaks during the reparation of the broken one. Therefore, the grid owner does not take any actions as long as at least one of the cables is working. When both are broken, there is an interruption that is estimated to cost the grid owner 300 000 SEK. Each cable is supposed to have a failure frequency of 0.1 failures/year.

a) How much does this strategy cost the company in average per year?

Suppose now that the grid owner is considering a new strategy, where a broken cable is repaired immediately. At every reparation, there is however a risk of 10 %, that the unbroken cable is destroyed by the reparation work. Despite the new strategy, there is still the risk that both cables break before the reparation is started. Suppose that this happens in average every 150 years.

b) Would a change of strategy be economically profitable?

c) The estimation of the risk that an unbroken cable is destroyed at reparation of the other cable is very uncertain. How high must that risk be, in order to make the two strategies equally profitable? Suppose that the risk today is 20 %, but decreases by 1 % per year, due to improved technology. In how many years should the company change strategy?

2 Markov

Problem 2.1

Basic understanding

a) Complete the transition rate matrix. $Q=\begin{bmatrix} & 4 & 2 \\ 1 & -3 & \\ 0 & 1 & \end{bmatrix}$

b) Write the equation system for the steady-state distribution, and solve it.

Problem 2.2

A simple Markov example

Suppose that you have 2 components, component a and component b, which independently of each other can be in operation or out of operation. They have different and constant reparation frequencies and failure frequencies. Suppose that two events (reparation or failure) can not occur at the same time.

a) What do the state transition diagram and the transition rates for this system look like?
b) Which is the steady-state distribution for the different states?
c) Compute the availability of the system, for both a series connection and a parallel connection.

Problem 2.3

Some more states and questions

A system of two components in series fails when one of the components breaks. The components have the same constant failure rate and reparation rate, and when one of them breaks, they can not be repaired at the same time. The failure rates are for both components 1/500, and the reparation rates are 1/10.

a) How high is the asymptotic availability?
b) Calculate MTBF (Mean Time Between Failures) for this system.
c) How long is the expected time when both components are broken, between two visits in the state at which the system is working?
d) Suppose that one component is broken. How big is then the chance that this component is repaired compared to the chance that the other component breaks?

Problem 2.4

A generator example – comparison of discrete and continuous calculation

A generator has a normal state (in function). Studies have shown that this type of generator vibrates more strongly just before it is going to fail; this behavior is continuously monitored (condition monitoring). The method that is used for monitoring of these vibrations has an efficiency of 90 %, i.e., the probability that the vibrations are registered before a failure occurs is 90 %. The generator fails in average one every 200 days. In those cases where vibrations are detected, the generator is taken out of operation for five days, for maintenance. There is a certain probability that this maintenance is unsuccessful, which if it happens results in a failure. This risk is estimated to be 10 %. In case of a failure, the generator is taken out of operation for 30 days. After successful maintenance and reparations, the generator is supposed to return to its normal state.

a) Draw up the states and write the transition rates of the states. In order to calculate the transition rates, the following can be used: $\lambda_{ij}=Pr_{ij}\,\lambda_i$. Pr_{ij} is the transition probability from i to j, and λ_i is the transition rate out of state i.
b) Explain what the transition rate matrix looks like, and write the equations which determine of the steady-state distribution.
c) Suppose that we use a discrete markov process. What do now the state transition diagram and the transition probabilities look like? (Suppose that a jump is made every day, and do not forget that a jump can go back to the same state from where it parted. The sum of all transition probabilities out of each state shall be equal to 1.)
d) Write the transition matrix and the equations for the stationary distribution.
e) Evaluate and compare the equation systems in b) and d).

Problem 2.5

Finding Q-matrix and illustrating transitions for systems with many states

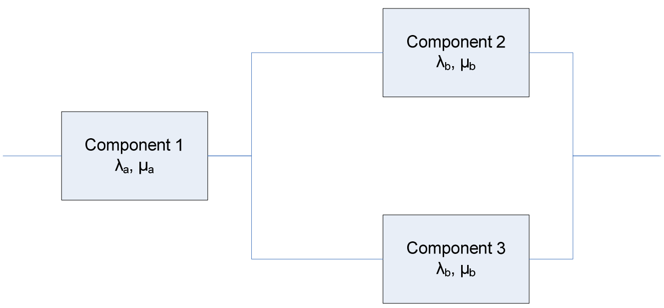
An electric cable is supposed to break down in two steps, before it finally fails. If the cable fails, a reparation making the cable as good as new takes time t_r. If no reparations are made, the cable goes from new to broken in time t_l. Between the states ”new” and ”broken”, there are two states of increasing decay, which have to be passed. The transition rate between two states is supposed to be the same in both directions. Maintenance is done with rate μ_u from the two states of decay. No maintenance is done if the cable is as good as new. The maintenance can have as result that the cable becomes as good as new, or that the cable goes back to the stat at which it was when the maintenance begun. Maintenance requires time t_u and the probability that the cable after maintenance becomes as good as new is X if it comes from the first state of decay, and Y if it comes from the second state of decay. (All transition rates and times have the unit days.)

a) Draw up the states and transition rates. (There are six states)
b) Write the transition rate matrix

Problem 2.6

Another example of a system with many states

In the connection diagram below, components 2 and 3 are identical. In case of a system failure, no more components can break. Only one component can be repaired at the time, and when more then one component needs reparation, the more important one is chosen.



a) Define states. (There are five states.)
b) Draw up state transition diagram with transition rates.
c) Derive the transition rate matrix.

d) Given that λ_a= 1/100, μ_a= 1/5, λ_b= 1/50, μ_b= 1/10, the steady-state is π_1= 0.645, π_2=0.258 π_3= 0.32 π_4= 0.052 π_5= 0.013, where the states are:
State 1: all components are working
State 2: component 2 or component 3 has failed
State 3: component 1 has failed
State 4: components 2 and 3 have failed
State 5: component 1 and one of components 2 and 3 have failed

What is the asymptotic availability? If the process enters state 1, how long is the expected duration of the visit in this state?

Problem 2.7
A bicycle example of Markov!
A person, who regularly goes by bike, is asking you to help him estimate the costs of this habit. The starting point is when he just bought a new bike. The price of the new bike was 8000 SEK, but this cost shall not be included in the calculations; these shall only include future costs. The person wants an estimated average cost per year.

The person has no idea of how often, measured in time, he normally changes bikes, or how often he hands it in for reparation. On the other hand, he is very good at estimating the probability for him to do these things, given the state of the bicycle. He has been trying to estimate this in frequency per six months, in order to facilitate Markov calculations.

Just after it has been bought, the bicycle is in state ”new”. If during six months, the bicycle is more than only slightly used, it can no longer be considered as ”new”. The intensity for this to happen during a time of six months is 0.95; the only things that could lead to such a low use of the bike, are a longer time of illness, or extremely bad weather. Just after being “new”, the bicycle is always considered to be still in good condition.

When the bicycle is in a good condition, it has a transition rate of 0.20 times per six months to go to a state where it is in a slightly worse condition. The corresponding transition rate for the owner to become tired of his bicycle, sell it for 3000 SEK and then buy a new one for 8000 SEK is 0.05. The rate for the bicycle to break is 0.05 per six months.

When the bicycle is in a less good condition, the transition rate during six months for it to be handed in for maintenance, which costs 500 SEK, is 0.4, and after which the bicycle is assumed to be back in a good condition. The corresponding rate for the owner to become tired of his bicycle, sell it second-hand for 1000 SEK and then buy a new one for 8000, is 0.10, which is the same as the intensity for the bicycle to break. Otherwise, the bicycle remains whole, but in a less good condition.

If the bicycle is broken, the transition rate for the owner to throw or give it away and buy a new one for 8000 SEK, is 0.20 per six months (he is to lazy to try to get money for it). The transition rate that the owner has the bicycle repaired and fixed up for 1000 SEK, which brings the bicycle back to be in a good condition, is 0.60 per six months. Sometimes, the owner does not have time to take measures, and leaves the bicycle broken for a shorter time.

a) Define states, illustrate how they are connected, and write the transition probabilities of a Markov process.

b) Calculate the mean time for the bicycle to be in each state, how often it is whole, and mean time between buys of a new bicycle.
c) Estimate the mean annual cost of the owning and using of a bicycle. How much part of the total cost is made up by maintenance, reparation, buy of new bicycle and selling, respectively? Other smaller costs are included in the remaining costs (for example the buying of bicycle lock).

3 RCM and LCC

Problem 3.1
LCC analysis of new investments in a power network
Your task is to perform an LCC analysis applied to the power network in Problem 1.6. The following investments shall be analyzed:

1. Continue as before
2. Replacing line by cable
3. Possibility to feed the system from different directions
4. Changing of disconnecting switches to switchers

The table below shows the costs that are associated to each investment. For simplicity, we suppose that the investment is done at year 0, and that there are no fiscal effects. Preventive maintenance is carried out every year, and all alternatives have a life time of 30 years. The rest value of the investments at the end of year 30 is 20 % of the disposal cost. For the calculation of the annual cost for failure, SAIDI and SAIFI are used:

Failure cost per year= a*SAIDI + b*SAIFI, where the costs a and b are given in the table

A discount rate of 7 % shall be used. Try also what happens if you use 5 % and 9 % discount rate.

Costs	Do nothing	Replacing line by cable	Double feeding	Switchers	Cost SAIDI/SAIFI
Investment [SEK]	0	300 000	175 000	200 000	
Increase preventive maintenance [SEK/year]	0	0	4 000	4 000	
SAIDI*	0.55	0.25	0.38	0.36	20 000
SAIFI*	0.32	0.11	0.18	0.13	70 000

*Are not identical to the correct solution of Problem 1.6.
For the calculation of the present value of an annual cash flow, the cash flow is multiplied by ”sum present value factor”. r is the discount rate and n is the number of years.

$$\text{Present sum factor} = \frac{(1+r)^n - 1}{r(1+r)^n}$$

$$\text{Present value factor} = \frac{1}{(1+r)^n}$$

Problem 3.2
Example with a bicycle! As well LCC (part b) as RCM (part a).

a) Ru Lá Fort recently bought a bicycle. He is going to use it when going to work, and for cycling in the forest in his spare time. Ru Lá Fort would like to use his bicycle during all the year, except when it is very cold or too much snow. He has heard of RCM and wants to use that for his bike. He is now asking you to perform an RCM analysis. Make necessary assumptions.

b) Now assume instead that Ru Lá Fort comes to you before he buys his bicycle. He usually wears down his bicycles quite rapidly, and wonders if he should buy a cheap bicycle of lower quality or a more expensive one of higher quality.

   Ru has looked up two bicycles, of which he will choose one, and about these two he has collected some information. The more expensive one costs 7000 SEK, whereas the less expensive one costs only 2000 SEK. Since Ru often goes by bike, and since he uses it in a way that wears it down, he thinks that the cheap bike would hold for three years, and the expensive one for nine years. When the bike is broken, Ru can sell it to his friend Lett Lu Rad for 300 SEK, the same price no matter which of the bikes it is. Ru expects the costs for maintenance of the expensive bike to be SEK 300 the first year, and then to increase by 10 % each year. The cheaper one requires cheaper maintenance, which would cost SEK 150 per year. Ru is also a bit vain: he finds it embarrassing to go around with a worn bicycle. He has tried to estimate a cost corresponding to how much he will be embarrassed. The expensive bike will make him feel embarrassed for SEK 150 the first year, and then this cost will increase by 20 % per year. The cheaper one will make him feel ashamed for SEK 400 the first year, and then this cost will also increase by 20 % per year. Ru uses a discount rate of 25 %. Which alternative should he choose, considering economical aspects? The information is summarized below.

	Cheap bike	Expensive bike
Investment [SEK]	3 000	6 000
Maintenance [SEK]	300	300
Annual increase of maintenance	0 %	10 %
Price of embarrassment [SEK]	400	150
Annual increase of price of embarrassment	20 %	20 %
Life time [years]	3	9
Rest value [SEK]	-300	-300
Discount rate	25 %	25 %

   Tip: A cash flow CF year one, growing by g per year during t years, and being discounted by r per year, has the present value

$$SPV(r,g,t,CF) = \frac{CF}{r-g}\left(1-\frac{(1+g)^t}{(1+r)^t}\right)$$

   A cash flow CF occurring in t years has, if the discount rate is r, the present value

$$PV(r,t,CF) = \frac{CF}{(1+r)^t}$$

Problem 3.3

LCC for continuous monitoring of PD

A director of the electricity company Wattvolt AB has heard of a new sort of condition monitoring of electric cables. At present, the company is using partial discharge (PD) - measuring offline. The method determines the location on the cable where it has been damaged. The disadvantage of this method is that measurements are done only during a limited time, and that the electric power must be switched off during the measurements. The new PD method is online, and thus measures all of the time, which leads to a higher reliability. The produced information is continuously analyzed by a computer at a control center. The electricity company might find it interesting to install a continuous measurement of the most important parts of their grid.

Investigate whether it would be profitable to purchase and use a continuous PD monitoring. The costs of continuous PD are: the investment cost the first year, the monitoring every year and preventive maintenance of the equipment every 10 years. The equipment is estimated to last for 30 years, where after it will be scraped, which brings a scraping bonus. Of those 0.03 failures/kilometer and year which can be discovered by the PD technique, approximately 90 % could be prevented with KPD, and 20 % with offline measurement. At present PD controls offline are done every third year. The following data are available for both alternatives.

Disregard fiscal effects and make a judgment of whether continuous condition monitoring should be used. The discount rate is 7 %.

PD data

Entry	Online	Offline
Investment [SEK]	17 000 000	0
Monitoring [SEK/year]	100 000	0
Preventive maintenance of equipment for continuous PD [SEK]	1 000 000	0
Interval between maintenance of equipment for continuous PD [years]	10	0
Scheduled PD controls offline (including interruption costs) [SEK]	0	1 000 000
Interval between scheduled PD controls [year]	0	3
Rest cost of PD online equipment after 30 years [SEK]	-500 000	0
Grid length [km]	100	100
Interruption cost when failure occurs [SEK]	500000	500000
Failure per kilometer and year, which could be discovered by PD	0,03	0,03
Part of the failures showing PD, that could be prevented	90 %	20 %

4 Network Performance Assessment Model (NPAM)

Problem 4.1
What part of the electricity business is affected by the Network Performance Assessment Model, and why has that particular part been chosen for regulation?

Problem 4.2
Assume that an electrical distribution network has an expected interruption cost of SEK 100 000, and a maximum cost reduction of reliability of SEK 1 000 000 from the NPAM. Sketch a graph showing the reduction (and possibly addition) of reliability cost obtained by the company, as a function of its reported interruption cost. What are the highest and the lowest potential reduction of reliability cost?

Problem 4.3
Give at least three examples of data that the reference network of the NPAM and the real distribution network have in common. What are these data usually called, and what was the basic idea of the inventor in his choice of these data?

Problem 4.4
The reference network of the NPAM is radial. What does that mean? Does the NPAM take into consideration that real grids do not always look like that, and if so, in what way?

Problem 4.5
Explain the meaning of immediate surroundings in the algorithm of the NPAM.

Problem 4.6
What is the name of the reliability method on which some of the template functions in the NPAM are based (for example redundancy)? Explain shortly the difference between that method and analytical reliability methods. Write at least three simplifications that have been done during these calculations.

Problem 4.7
Rank the following four customers according to the interruption cost that they cause the net owner when an interruption occurs (the same interruption length for all, and assuming that they all have their own subscription): A large villa close to Uppsala, a single apartment in the city of Stockholm, a large paper mill in Norrland 10 km from the next village, and a summer house in the archipelago of Stockholm.

Problem 4.8
What is the formula of the debiting grade? When does the net owner risk a further inspection and possible measures from Energy Markets Inspectorate (Energimarknadsinspektionen)?

5. Solutions

5.1 Solutions to reliability calculations for power networks

Problem 1.1

a) A primary failure is when a component fails independently of the other components in the system, for example as a result of wear or external effects such as falling trees. Secondary failures are those caused by other failures somewhere else in the system, for example caused by overload or by a circuit breaker that does not open in a situation where it could stop a failure from causing further effects on the system.
b) A failure on parts or the whole of the system, caused by a failure in only one component, (for example in a series connection) is a failure of the first order. If it takes two components to fail at the same time, in order to cause a partial or complete system failure, it is a failure of the second order (for example in a parallel connection); a failure of the third order requires for three components to fail, and so on.
c) Component redundancy means that one or several components are doubled or multiplied. System redundancy is when there is at least one parallel reserve system. In a power system, redundancy can also be created for example by introducing an alternative feeding to the system, or parts of it, which means that the system is fed by electricity in more than one node.
d) An example of active redundancy is when two parallel cables are in operation at the same time, and get disconnected immediately by a circuit breaker when a failure occurs. An example of passive redundancy is a reserve cable which normally is not in operation, but which by failure can be manually connected so the system. A disadvantage of active redundancy is that the wear on the reserve component is higher if it is in constant operation, and that the required automatic circuit breaker can be expensive (and is yet another source of failure).
e) x*y kWh/year and customer.
f) y/x failure/year and customer.
g) ASUI = 1-ASAI = 0.0001 (unavailability). There are 365*24 =8760 hours in one year, which means a total length of interruption of 8760*0.001 = 0.876 h/year = SAIDI. Since all failures last for one hour, we have SAIFI = 1*SAIFI = 0.876 failures/year.
h) CAIDI is a measure of average length of interruption. If an interruption that lasts less than the average interruption time of the system is eliminated, SAIFI as well as SAIDI decreases, but the average interruption length, CAIDI, increases: CAIDI = SAIDI/SAIFI, so in those cases when SAIFI decreases more that SAIDI, CAIDI increases.
i) A state where the system always is in working order if the component is in working order, and where the system fails if the component fails in this state.

Problem 1.2

a) Minimal paths: {1} and {2 3}. Minimal cuts: {1 2} and {1 3}.
b) If 1 is in function, the system is always in function and therefore equal to 1. If 1 is not in function, the minimal cuts are {2} and {3}. The pivotal formula is: $\theta(\mathbf{X}) = x_1*\theta(1_1,\mathbf{X}) + (1- x_1)*\theta(0_1,\mathbf{X}) = x_1*1 + (1- x_1)* x_2* x_3 = x_1 + x_2* x_3 - x_1*x_2*x_3$. The function probability of the system is obtained by entering the probabilities for the components in the fundamental form of the structure function: $\theta(\mathbf{P}) = 0.999 + 0.998*0.997 - 0.999*0.998*0.997 = 0.999995006$.

Problem 1.3

a) $\beta(i) = \eta(i) / 2^{n-1}$, where $\eta(i)$ is the number of critical states for component nr i, and n is the total number of components in the system ➔ $\beta(i) = \eta(i) / 4$. Component 1 has two critical states (two or three broken, the other ones whole), while components 2 and 3 have one critical state each. $\beta(1) = 2/4 = 0.5$, $\beta(2) = \beta(3) = 1/4 = 0.25$. **It will therefore be prioritized to first improve component 1, and after that one of the other two.**

b) $I^{\beta}(i{:}t)$ is the derivative of the structure function with respect to component i, with the function probabilities put into the function:
$I^{\beta}(1{:}t) = 1 - p_2 * p_3 = 1-0.998*0.997 = 0.0049940$,
$I^{\beta}(2{:}t) = p_3 - p_1 * p_3 = 0.997-0.999*0.997 = 0.0009970$ and
$I^{\beta}(3{:}t) = p_2 - p_1 * p_2 = 0.998-0.999*0.998 = 0.0009980$.
➔ **The order of prioritizing is: Component 1, then 3 and then 2.** Indicates how sensitive the function probability of the system is to a small change of the function probability for the components.

c) $I^{CR}(i) = \dfrac{I^{\beta}(i:t)*(1-p_i(t))}{1-h(\boldsymbol{p}(t))}$, where h(**p**(t)) is the probability function of the system ➔

$$I^{CR}(1) = \frac{0.0049940*(1-0.999)}{1-0.999995006} = 1.00000$$

$$I^{CR}(2) = \frac{0.000997*(1-0.998)}{1-0.999995006} = 0.39928$$

$$I^{CR}(3) = \frac{0.000998*(1-0.997)}{1-0.999995006} = 0.59952$$

The calculations above are made without rounding of each partial calculation of Birnbaum's measure and the function probability of the system.

➔ **The order of prioritizing is: Component 1, then 3 and then 2.**

d) Vesely-Fussel $I^{VF}(i)$ is the probability that component i is included in the minimal cut causing a possible failure. Component 1 in part of all minimal cuts, so $I^{VF}(1) = 1.00$ (100 %). The minimal cut of the system is: {1 2} and {1 3}. The probability for the cut where component 2 is included is therefore
$(1-p_2)/((1-p_2)+ (1-p_3)) = 0.4$ (40 %) ➔
$I^{VF}(2) = 0.4$ and $I^{VF}(3) = 1-0.4 = 0.6$ (60 %).

➔ **The order of prioritizing is: Component 1, then 3 and then 2.**

Problem 1.4

	A			B			C		
	λ	**r**	**U**	**λ**	**r**	**U**	**λ**	**r**	**U**
A	1	2	2	1	0.5	0.5	1	0.5	0.5
B	1	0.5	0.5	1	2	2	1	2	2
C	0.1	0.5	0.05	0.1	2	0.2	1	2	2
Sum	2.1	-	2.55	2.1	-	2.7	3	-	4.5

With an average of 1 failure per year for C, and a probability of 10 % that the switcher fails, the average failure frequency for A and B is 0.1 failures/year.

$$SAIFI = \frac{2.1*(0.4+0.3)+3*0.3}{1} = 2.37 \text{ failures/year}$$

$$SAIFI = \frac{2.55*0.4+(2.7+4.5)*0.3}{1} = 3.18 \text{ hours/year}$$

$$ASAI = \frac{1*8760-(2.55*0.4+(2.7+4.5)*0.3)}{1*8760} = \frac{8760-3.18}{8760} = 0.9996370 \text{ (Availability)}$$

$$ASUI = 1 - ASAI = 0.000363 \text{ (Unavailability)}$$

Problem 1.5

a) Without approximation: 0.005980994. With the approximation of neglecting the cut of all components: 0.005981; an overestimation of around 1 ppm. Neglecting all cuts: 0.006; an overestimation of around 0.184 %.

b) Write an equation. The exact formula gives a lower value then the approximate one, since it contains a division with a value greater than one. Its lowest allowed value is therefore 0.999 *[the exact value of the formula]:

$$\frac{(\lambda_x^{\max} * \lambda_y^{\max})*(r_x + r_y)}{1+\lambda_x^{\max} * r_x + \lambda_y^{\max} * r_y} = 0.999*(\lambda_x^{\max} * \lambda_y^{\max})*(r_x + r_y) \to [given] \to$$

c) $$\frac{(3*\lambda_y^{\max})*(\frac{4}{8760})}{1+2*\lambda_y^{\max} * \frac{2}{8760} + \lambda_y^{\max} * \frac{2}{8760}} = 0.999*(3*\lambda_y^{\max})*(\frac{4}{8760}) \to$$

$$\frac{1}{0.999} - 1 = \lambda_y^{\max} * \frac{6}{8760} \to \lambda_y^{\max} = \frac{8760}{6}\left(\frac{1}{0.999} - 1\right) \approx 1.46; \ \lambda_x^{\max} = 2*\lambda_y^{\max} \approx 2.92$$

The RADPOW equation is valid for failure frequencies of magnitude up to one failure per year and component, if the exactness has to be at least 0.1 %. Most components in an electrical system failure more often than that, so usually the equation has a greater exactness than 0.1 %.

d) Can be computed directly by a small modification of the equation from b):
$$\lambda_y^{\max} = \frac{8760}{6}\left(\frac{1}{0.9999} - 1\right) \approx 0.146;\ \lambda_x^{\max} = 2 * \lambda_y^{\max} \approx 0.292$$
$$\lambda_y^{\max} = \frac{8760}{6}\left(\frac{1}{0.99} - 1\right) \approx 14.75;\ \lambda_x^{\max} = 2 * \lambda_y^{\max} \approx 29.5$$

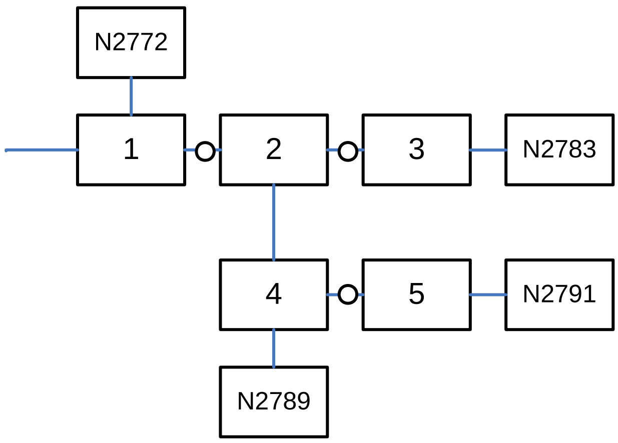
0.146 and 0.292 failures/year are common magnitudes of component failures in an electrical system, so the approximation often leads to an error of around 0.01 %. That tens of failures occur every year is a lot for most technical systems, so errors larger than 1 % (due to approximations) should be rare.

e) If the entire derivation in b) is studied, it can be concluded that: $\lambda_y^{\max} = \frac{8760}{z}\left(\frac{1}{0.9999} - 1\right)$; z = 2+2*[the difference in failure frequency for x and y. These are here 1 and 20.]
$$\lambda_y^{\max} = \frac{8760}{4}\left(\frac{1}{0.999} - 1\right) \approx 2.19;\ \lambda_x^{\max} = \lambda_y^{\max} \approx 2.19$$
$$\lambda_y^{\max} = \frac{8760}{22}\left(\frac{1}{0.999} - 1\right) \approx 0.399;\ \lambda_x^{\max} = 20 * \lambda_y^{\max} \approx 7.98$$

General conclusion: The lower the failure frequency, the smaller the error from the approximate formula. A somewhat higher failure frequency of one of the components can be accepted, if the other one has a several times lower failure frequency, with maintained accuracy.

Problem 1.6

a) SAIFI [failure per year and customer] is not affected since the same customers will have the same failures in both cases. Disconnecting switchers will only shorten the average length of failure, which affects for example SAIDI, CAIDI and AENS, but not SAIFI.
b) Can look in different ways, one example:



Note! The model above is not equivalent to the real grid structure, but describes correctly the course of failure. Also other models could do that.

Hint 1: Failure frequency = 0.680*0.02 = 0.01360 failures/year
Hint 2: Failure frequency = 0.818*0.02 = 0.01636 failures/year
Hint 3: Failure frequency = (0.519 + 0.039)*0.2 = 0.11160 failures/year
Hint 4: Failure frequency = 0.345*0.02 = 0.00690 failures/year
Hint 5: Failure frequency = (0.700 + 0.233)*0.2 + 8*0.02 = 0.18676 failures/year
Disconnecting Switch: Between (N2772 and 1 – has no importance), 1 and 2, 2 and 3, 4 and 5 (in the figure a ring).
Fuse: Between 1 and 2 (not in the figure).
c)

Grid station:	**N2772**			**N2783**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	2.5	0.034	0.01360	2.5	0.034
2	0.009816	1	0.009816	0.01636	2.5	0.0409
3	0.06696	1	0.06696	0.11160	2.5	0.279
4	0.00414	1	0.00414	0.00690	2.5	0.01725
5	0.112056	1	0.112056	0.18676	1	0.18676
In total:	**0.256572**	**-**	**0.376972**	**0.38522**	**-**	**0.70791**
Grid station:	**N2789**			**N2791**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	2.5	0.034	0.01360	2.5	0.034
2	0.01636	2.5	0.0409	0.01636	2.5	0.0409
3	0.11160	1	0.11160	0.11160	1	0.11160
4	0.00690	2.5	0.01725	0.00690	2.5	0.01725
5	0.18676	1	0.18676	0.18676	2.5	0.4669
In total:	**0.38522**	**-**	**0.54051**	**0.38522**	**-**	**0.82065**

$$SAIFI = \frac{51 * 0.256572 + (33 + 10 + 13) * 0.38522}{51 + 33 + 10 + 13} = 0.3239 \text{ failures/year}$$

$$SAIDI = \frac{51 * 0.376972 + 33 * 0.70791 + 10 * 0.54051 + 13 * 0.82065}{107} = 0.5482 \text{ hours/year}$$

$$CAIDI = \frac{SAIDI}{SAIFI} = 1.6925 \text{ hours/failure}$$

$$AENS = \frac{349532 * 0.376972 + 183009 * 0.70791 + 142837 * 0.54051 + 74220 * 0.82065}{107 * 8760} = 0.4291$$

kWh/year.

c) Hints 1, 2 and 4 remain unchanged. In hint 3, the failure frequency decreases 10 times to 0.01116 failures/year. In hint 5, the failure frequency decreases approximately 10 times, to 0.018676 failures/year.

Grid station:	**N2772**			**N2783**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	2,5	0.034	0.01360	2.5	0.034
2	0.009816	1	0.009816	0.01636	2.5	0.0409
3	0.006696	1	0.006696	0.011160	2.5	0.0279
4	0.00414	1	0.00414	0.00690	2.5	0.01725
5	0.0112056	1	0.0112056	0.018676	1	0.018676
In total:	**0.0954576**	**-**	**0.2158576**	**0.116696**	**-**	**0.288726**
Grid station:	**N2789**			**N2791**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	2.5	0.034	0.01360	2.5	0.034
2	0.01636	2.5	0.0409	0.01636	2.5	0.0409
3	0.011160	1	0.011160	0.011160	1	0.01116
4	0.00690	2.5	0.01725	0.00690	2.5	0.01725
5	0.018676	1	0.018676	0.018676	2.5	0.04669
In total:	**0.116696**	**-**	**0.271986**	**0.116696**	**-**	**0.3**

$$SAIFI = \frac{51 * 0.0954576 + (33 + 10 + 13) * 0.116696}{51 + 33 + 10 + 13} = 0.1066$$ failures/year (32.9 % of the original)

$$SAIDI = \frac{51 * 0.2158576 + 33 * 0.288726 + 10 * 0.271986 + 13 * 0.3}{107} = 0.2538$$ h/year (46.3 %)

$$CAIDI = \frac{SAIDI}{SAIFI} = 2.3809 \text{ h/failure (140.7 \%)}$$

$$AENS = \frac{349532 * 0.2158576 + 183009 * 0.288726 + 142837 * 0.271986 + 74220 * 0.3}{107 * 8760} = 0.2021$$ kWh/year (47.1 %).

d) As in c), except that all elements having a repair time longer than 1 hour have been taken away:

Grid station:	**N2772**			**N2783**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	2.5	0.034	0.01360	2.5	0.034
2	-	-	-	0.01636	2.5	0.0409
3	-	-	-	0.11160	2.5	0.279
4	-	-	-	0.00690	2.5	0.01725
5	-	-	-	-	-	-
In total:	**0.0636**	**-**	**0.184**	**0.19846**	**-**	**0.52115**
Grid station:	**N2789**			**N2791**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	2.5	0.034	0.01360	2,5	0.034
2	0.01636	2.5	0.0409	0.01636	2,5	0.0409
3	-	-	-	-	-	-
4	0.00690	2.5	0.01725	0.00690	2,5	0.01725
5	-	-	-	0.18676	2,5	0.4669
In total:	**0.08686**	**-**	**0.24215**	**0.27362**	**-**	**0.70905**

$$SAIFI = \frac{51*0.0636+33*0.19846+10*0.08686+13*0.27362}{107} = 0.1329 \text{ failures/year (41.0 \%)}$$

$$SAIDI = \frac{51*0.184+33*0.52115+10*0.24215+13*0.70905}{107} = 0.3572 \text{ h/year (65.2 \%)}$$

$$CAIDI = \frac{SAIDI}{SAIFI} = 2.6877 \text{ hours/failure (158.8 \%)}$$

$$AENS = \frac{349532*0.184+183009*0.52115+142837*0.24215+74220*0.70905}{107*8760} = 0.2634$$

kWh/year (61.4 %).

e)

Grid station:	N2772			N2783		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	1	0.01360	0.01360	1	0.01360
2	0.009816	1	0.009816	0.01636	2.5	0.0409
3	0.06696	1	0.06696	0.11160	2.5	0.279
4	0.00414	1	0.00414	0.00690	2.5	0.01725
5	-	-	-	-	-	-
In total:	**0.144516**	**-**	**0.244516**	**0.19846**	**-**	**0.50075**
Grid station:	**N2789**			**N2791**		
Failure in:	**λ (failures/year)**	**r (h)**	**U (h/year)**	**λ (failures/year)**	**r (h)**	**U (h/year)**
Grid station	0.05	3	0.15	0.05	3	0.15
1	0.01360	1	0.01360	-	-	-
2	0.01636	2.5	0.0409	-	-	-
3	0.11160	1	0.11160	-	-	-
4	0.00690	2.5	0.01725	-	-	-
5	-	-	-	0.18676	2.5	0.4669
In total:	**0.19846**	**-**	**0.33335**	**0.23676**	**-**	**0.6169**

$$SAIFI = \frac{51 * 0.144516 + 43 * 0.19846 + 13 * 0.23676}{107} = 0.1774 \text{ failures/year (54.8 \%)}$$

$$SAIDI = \frac{51 * 0.244516 + 33 * 0.50075 + 10 * 0.33335 + 13 * 0.6169}{107} = 0.3771 \text{ h/year (68.8 \%)}$$

$$CAIDI = \frac{SAIDI}{SAIFI} = 2.1257 \text{ h/failure (125.6 \%)}$$

$$AENS = \frac{349532 * 0{,}244516 + 183009 * 0{,}50075 + 142837 * 0{,}33335 + 74220 * 0{,}6169}{107 * 8760} = 0{,}2886$$

kWh/year (67.3 %).

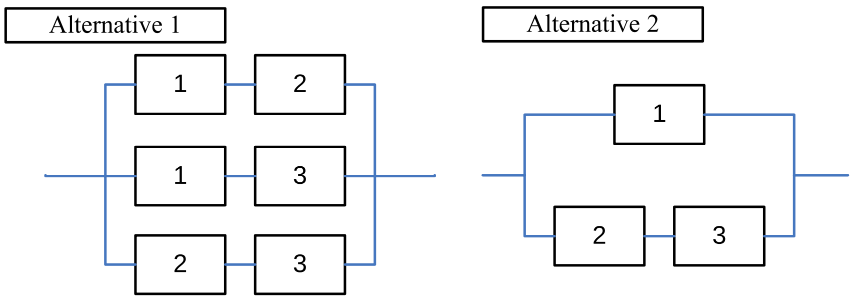
f) In this case, underground line gives the best result, which is due to the fact that for this system, the majority of failures occur in the overhead line. However, digging down 1.5 km of cable can be a comparatively expensive measure, depending on the structure of the terrain. Buying automatic switchers is a somewhat better solution than introducing a second feeding. The second feeding could give a better result, though, if it is combined with other investments such as more disconnecting switchers or circuit breakers. All actions give, however, a good decrease of the number of failures, all of them around a halving. Therefore, when choosing between these alternatives, a lot of consideration should be given to the investment costs, and many times it would probably be wise to invest in a combination of these alternatives. In the example, several approximations have also been made, which can have effect: underground line usually has a longer repair time than overhead line, the ratio between the failure frequencies for overhead line and underground line looks differently for different terrains (forests, open landscapes), the introduction of new components means in reality new failure sources, and the part of the system that we are looking at is of course dependent of the rest of the network.

Problem 1.7

Call the new line 1, and the two older 2 and 3.

a) 1: The minimal paths and the minimal cuts are in this case the same: {1 2}, {1 3} and {2 3}.
   2: The minimal paths are {1} and {2 3}, the minimal cuts are {1 2} and {1 3}.

b) Models can be made either by making a parallel connection of minimal paths (where the components within each minimal path are connected in series), or by connecting in series the minimal cuts (where the components within each minimal cut are connected in parallel) – these both models give the same results for the reliability calculations, both alternatives are correct. Here is a figure of the alternatives:



c)

$$\theta(X)_1 = 1-(1-x_1*x_2)*(1-x_1*x_3)*(1-x_2*x_3) = 1-(1-x_2*x_3-x_1*x_3-x_1*x_2+$$
$$x_1*x_2*x_3^2+x_1*x_2^2*x_3+x_1^2*x_2*x_3-x_1^2*x_2^2*x_3^2) = \{x_*^2 = x_*\} =$$
$$x_2*x_3+x_1*x_3+x_1*x_2-2*x_1*x_2*x_3$$
$$\theta(X)_2 = 1-(1-x_1)*(1-x_2*x_3) = x_1+x_2*x_3-*x_1*x_2*x_3$$

d) Put x_1=x_2=x_3=p in the fundamental form of the two functions: $P_1 = 3*p^2-2*p^3$ and $P_2 = p+p^2-p^3$.

e) If you let p=0,99 in the expressions from d), you get probability 0,999702 for alternativ 1, det billigare alternativet ger systemfunktion och sannolikheten 0,999801 för det andra dyrare alternativet. The difference in probability is 0,999801 - 0,999702 = 0,000099. In one year, that makes a difference of 365*24*0,000099 = 0,86724 hours and a difference in interruption cost of SEK 0,8672*100 000 = 86 724. It is thus economical to use only the cheaper alternative, if it in average gives a capital cost that is at least 86 724 SEK lower per year than the more expensive alternative.

f) A constant failure frequency of 0.02 failures per year, gives the survival function $e^{-0.02*t}$. By inserting the components' survival functions in the structure function of the corresponding system, the survival function of the system is obtained. By integrating the survival function of the system from zero to infinity, the mean time to failure (MTTF) is obtained:

For the cheaper alternative:

$$R_1 = 3 * e^{-0.04*t} - 2 * e^{-0.06*t}$$

$$\int_0^\infty R_1 = \left[3*(-25)*e^{-0.04*t} - 2*(-16\frac{2}{3})*e^{-0.06*t} \right]_0^\infty = 75 - \frac{100}{3} = 41\frac{2}{3}$$ years to failure.

For the more expensive alternative:

$$R_2 = e^{-0.02*t} + e^{-0.04*t} + e^{-0.06*t}$$

$$\int_0^\infty R_2 = \left[(-50)*e^{-0.02*t} + (-25)*e^{-0.04*t} - (-16\frac{2}{3})*e^{-0.06*t} \right]_0^\infty = 50 + 25 - 16\frac{2}{3} = 58\frac{1}{3}$$ years to failure.

So in average it takes (175-125)/3 = 50/3 = 16 + 2/3 years more for the expensive alternative, before a failure in the system occurs, which corresponds to an increase of 40 % of the estimated/calculated length to the first failure.

Problem 1.8

$$\text{MTTF}_{\text{parallel}} = \int_0^\infty R_{parallel} = \left[2*(-\frac{1}{\lambda})*e^{-\lambda*t} - (-\frac{1}{2\lambda})*e^{-2*\lambda*t} \right]_0^\infty = \frac{2}{\lambda} - \frac{1}{2\lambda}$$

If the failure frequency is 0.1 failures/year, the MTTF is 15 years for a parallel system that can not be repaired in case of a failure. Mean time to at least one failure in one of the cables can be calculated in the same way as MTTF for a series system:

$$\text{MTTF}_{\text{series}} = \int_0^\infty R_{series} = \left[(\frac{1}{2\lambda})*e^{-2*\lambda*t} \right]_0^\infty = \frac{1}{2\lambda}$$

If the failure frequency is 0.1 failures/year, MTTF is 5 years.

The survival function, R, is calculated using calculations of structure functions for a normal simple system of series and parallel connections, where the survival function of the corresponding component is inserted. The survival function is calculated according to the simplified formula for constant failure frequencies: $R = e^{-\lambda*t}$.

a) Mean time to system failure is in average 15 years. In case of an error, there is a cost of SEK 2*100 000 + 300 000 = 500 000. This means an average annual cost of 500 000 SEK / 15 years = 33 333 SEK / years.

b) Mean time to system failure as a result of failure in both components at the same time is 150 years. Mean time to failure in at least one component is 5 years. Every 150 years there is a cost as big as the one in a), i.e. SEK 500 000, which means SEK 3 333 per year. Every 5 years, there is 90 % of the times a cost of SEK 100 000, and 10 % of the times a cost of SEK 500 000. In average, this cost is SEK 140 000. Per year this is SEK 28 000. With this, the average annual cost is SEK 3 333 + SEK 28 000 = SEK 31 333. The proposed new strategy is therefore SEK 2 000 cheaper per year than the present strategy.

c) 3 333 SEK/year + x SEK/year = 33 333 SEK/year ==> x = 30 000 SEK/year;
y SEK/5 year = 30 000 SEK/year ==> y = 150 000 SEK.
100 000 * z + 500 000 * (1-z) = 150 000 ==> 400 000 z = 350 000 ==> z = 0.875.

If the risk is 12.5 %, the two strategies have the same profitability. At higher risks, the present strategy is more profitable, and at lower risks the new one. The company should, with the new assumption, change strategies in 7.5 years.

5.2 Solutions to Markov

Problem 2.1

a) In an intensity matrix, the sum of the elements in each row shall be equal to zero.

$$Q=\begin{bmatrix} -6 & 4 & 2 \\ 1 & -3 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

b) The asymptotic distribution is the solution to the equations

$\pi Q=0$

$\sum_i \pi_i = 1$

The vector π that fulfills these equations is equal to the probability distribution for the system at a time in the far future. In this case we have:

$$0=\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix}\begin{bmatrix} -6 & 4 & 2 \\ 1 & -3 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$1=\pi_1+\pi_2+\pi_3$$

The matrix equation written as a system of equations:

$$-6\pi_1+\pi_2=0$$
$$4\pi_1-3\pi_2+\pi_3=0$$
$$2\pi_1+2\pi_2-\pi_3=0$$
$$\pi_1+\pi_2+\pi_3=1$$

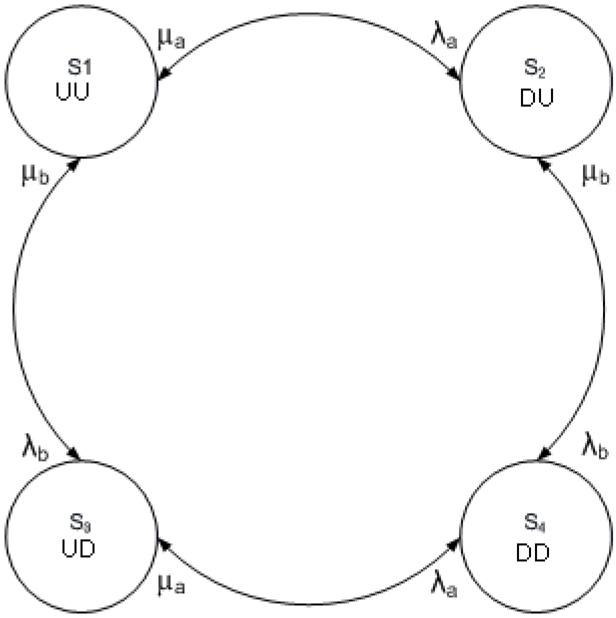
The solution to this equation system is: π_1=1/21, π_2=6/21, π_3= 14/21

Problem 2.2

a) There are 4 states:

State	Unit a (Up/Down)	Unit b (Up/Down)
S_1	U	U
S_2	D	U
S_3	U	D
S_4	D	D

Let λ be the failure intensity and μ the repair intensity. Then the transition diagram looks like this:



b) The intensity matrix is:

$$Q = \begin{bmatrix} -\lambda_a - \lambda_b & \lambda_a & \lambda_b & 0 \\ \mu_a & -\mu_a - \lambda_b & 0 & \lambda_b \\ \mu_b & 0 & -\mu_b - \lambda_a & \lambda_a \\ 0 & \mu_b & \mu_a & -\mu_b - \mu_a \end{bmatrix}$$

For the asymptotic solutions, the following equation system is used:

$\pi Q=0$

$$\sum_i \pi_i = 1$$

The answer is:

$$\pi_1 = \frac{\mu_a \mu_b}{(\lambda_a + \mu_a)(\lambda_b + \mu_b)}$$

$$\pi_2 = \frac{\lambda_a \mu_b}{(\lambda_a + \mu_a)(\lambda_b + \mu_b)}$$

$$\pi_2 = \frac{\lambda_b \mu_a}{(\lambda_a + \mu_a)(\lambda_b + \mu_b)}$$

$$\pi_2 = \frac{\lambda_a \lambda_b}{(\lambda_a + \mu_a)(\lambda_b + \mu_b)}$$

c) If the components are connected in series, both of them have to be in working order, i.e., the availability is π_1. If the components are connected in parallel, it is enough if only one component is working, i.e., $\pi_{1+}\ \pi_{2+}\ \pi_{3.}$

Problem 2.3

a)

Define the following states:

1: Both components are working

2: One component broken

3: Both components broken

the intensity matrix is:

$$Q = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\mu+\lambda) & \lambda \\ 0 & \mu & -\mu \end{bmatrix}$$

In state 1 two things can happen: Component 1 or component 2 could fail, both with intensity λ. In state 3, reparation is done only at one component at the time, which means that the intensity to go to state 2 is μ. For the calculation of the asymptotic availability, the following equations are used: $\pi Q=0$ and $\sum \pi = 1$.

Written as an equation system:

$$-2\lambda\pi_1 + \mu\pi_2 = 0$$
$$2\lambda\pi_1 - (\lambda+\mu)\pi_2 + \mu\pi_3 = 0$$
$$\lambda\pi_2 - \mu\pi_3 = 0$$
$$\pi_1 + \pi_2 + \pi_3 = 1$$

The solution to this system is:

$$\pi_1 = \frac{\mu^2}{\mu^2 + 2\mu + 2\lambda^2}$$
$$\pi_2 = \frac{2\lambda\mu}{\mu^2 + 2\mu + 2\lambda^2}$$
$$\pi_1 = \frac{2\lambda^2}{\mu^2 + 2\mu + 2\lambda^2}$$

With λ=1/500 and μ=1/10, we have π_1=0.961, π_2=0.038, π_3= 0.001

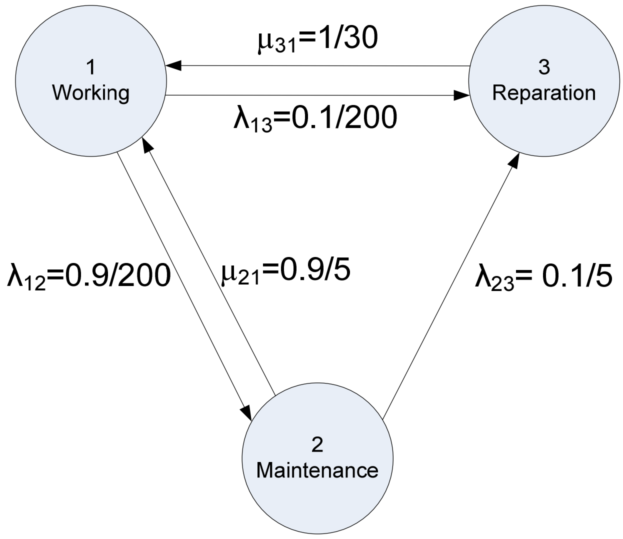
With this, the asymptotic availability for the series system is π_1=0.961.

b) The expected time between entrances in and exits from state *i* is obtained through $1/(\pi_i q_{ii})$. For the first working state, the time is 1/(0.961*2/500)=260 days.

c) The expected time during which both components have been broken is π_3*260 days = 0.26 days.

d) The probability for a broken component to be repaired is $q_{21}/q_{22} = \mu/(\mu+\lambda)$ = 98 %.

Problem 2.4



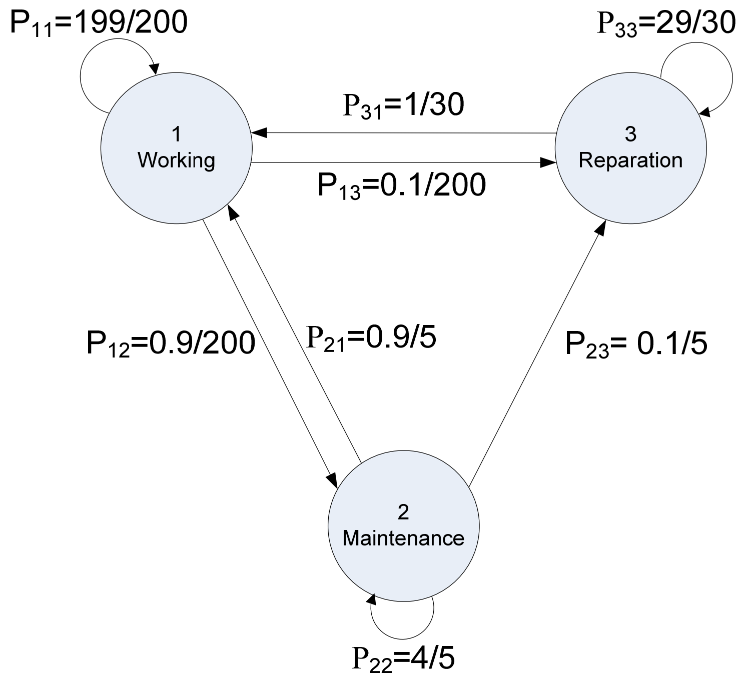
a) There are 3 states in total. By using the hint of multiplying the transition probabilities with the transition rate out of the state, the transition rates can be calculated. From state 3, the generator is always repaired and returns to a working state.

b) The steady state distribution of a continuous Markov process can be calculated from the following equation system: $\pi Q=0$ and $\sum_i \pi_i = 1$. The vector π fulfilling these equations is equal to the probability distribution for generator at a time in the far future. In this case we have:

$$\pi = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix}$$

$$Q = \begin{bmatrix} -1/200 & 0.9/200 & 0.1/200 \\ 0.9/5 & -1/5 & 0.1/5 \\ 1/30 & 0 & -1/30 \end{bmatrix}$$

c)



Suppose that there is one jump each day. The sum of all transition probabilities from one state must be equal to 1, including the probability to go back to the same state. From this, the transition probabilities for the generator can be calculated.

d) The steady-state distribution of a discrete Markov process can be calculated from the following equation system: sP=s and $\sum_i s_i = 1$. The vector s that fulfills these equations is equal to the probability distribution of the generator at a time in the far future. In this case:

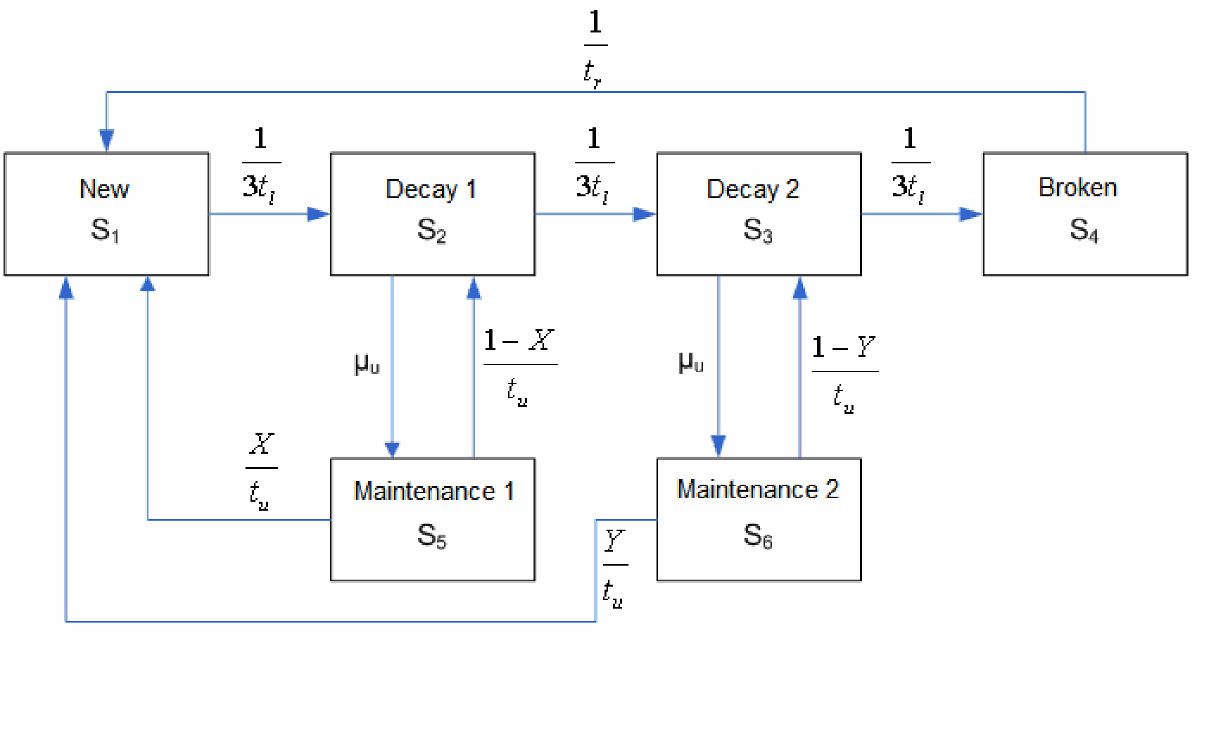
$$s = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}$$

$$P = \begin{bmatrix} 199/200 & 0.9/200 & 0.1/200 \\ 0.9/5 & 4/5 & 0.1/5 \\ 1/30 & 0 & 29/30 \end{bmatrix}$$

e) The equation systems in b) and in d) have the same solution:
π=s=[95.15 2.14 2.71]

Problem 2.5

a)



There are 4 states for the cable and 2 states for the maintenance. Every step towards in increased decay has an intensity of $1/(3t_l)$, since the total time between as good as new and failure mode is t_l and the intensity to go to the next state is the same for all states. From maintenance state 1, the cable can either go back to the same state from which it came, or become as good as new. The probability for the cable to become as good as new is X, and the time for maintenance is t_u. With this, the transition rates from the maintenance states can be determined.

b) The transition rate matrix looks like this:

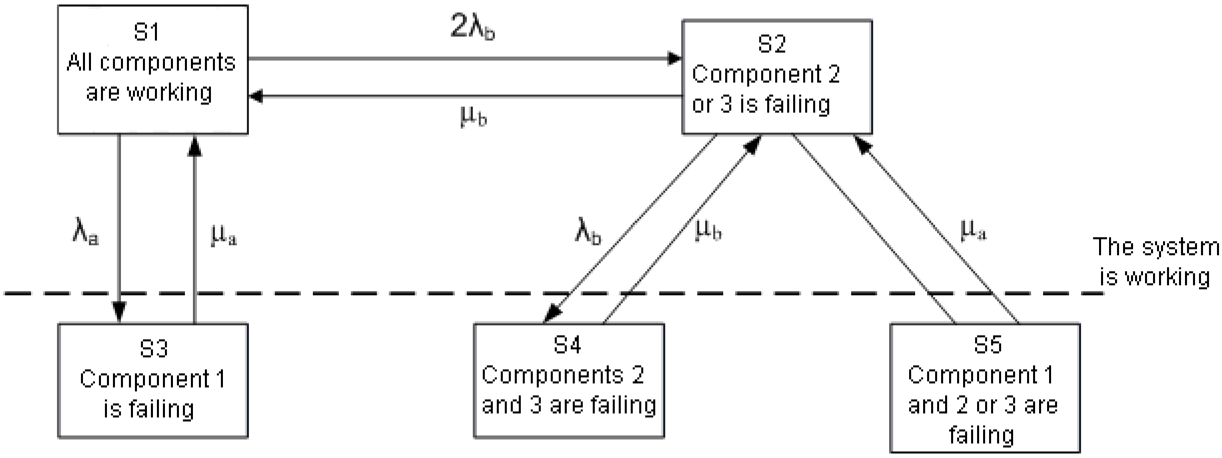
$$Q = \begin{bmatrix} -\frac{1}{3t_l} & \frac{1}{3t_l} & 0 & 0 & 0 & 0 \\ 0 & -\left(\frac{1}{3t_l}+\mu_u\right) & \frac{1}{3t_l} & 0 & \mu_u & 0 \\ 0 & 0 & -\left(\frac{1}{3t_l}+\mu_u\right) & \frac{1}{3t_l} & 0 & \mu_u \\ \frac{1}{t_r} & 0 & 0 & -\frac{1}{t_r} & 0 & 0 \\ \frac{X}{t_u} & \frac{1-X}{t_u} & 0 & 0 & -\frac{1}{t_u} & 0 \\ \frac{Y}{t_u} & 0 & \frac{1-Y}{t_u} & 0 & 0 & -\frac{1}{t_u} \end{bmatrix}$$

Problem 2.6

a) There are three components, and each component can be in two different states. This means that there are $2^3=8$ different combinations. Some of these are identical from a reliability point of view, and one is impossible since no more failures can occur when the system is out of working order. This corresponds to the last row in the table below. The different states can be defined according to the table.

State	Component 1	Component 2	Component 3	System function
S_1	1	1	1	1
S_2	1	1	0	1
	1	0	1	1
S_3	0	1	1	0
S_4	1	0	0	0
S_5	0	1	0	0
	0	0	1	0
	0	0	0	0

b) With the states given above, a state transition diagram can look like this:



c) This gives the transition rate matrix

$$Q = \begin{bmatrix} -(2\lambda_b + \lambda_a) & 2\lambda_b & \lambda_a & 0 & 0 \\ \mu_b & -(\mu_b + \lambda_a + \lambda_b) & 0 & \lambda_b & \lambda_a \\ \mu_a & 0 & -\mu_a & 0 & 0 \\ 0 & \mu_b & 0 & -\mu_b & 0 \\ 0 & \mu_a & 0 & 0 & -\mu_a \end{bmatrix}$$

d) The asymptotic availability is $\pi_1 + \pi_2 = 0.645 + 0.258 = 0.903$.
The transition rate out of state 1 is $2\lambda_b + \lambda_a$. The expected duration in this

Problem 2.7

a)

Time period: 6 months

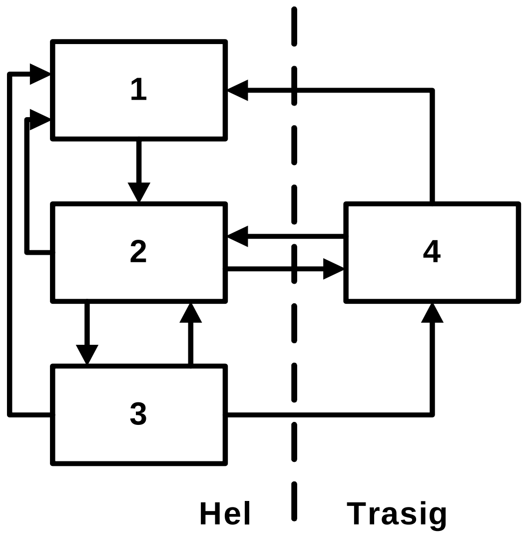
Number of states: 4

1: **As good as new**, The transition rate to state 2 during one period of time is 0,95.

2: **Good state**, transition probability of 0.05 to go to state 1, 0.20 to state 3 and 0.05 to go to state 4, during one period of time.

3: **Worse state**, transition probability of 0.10 to go to state 1, 0.40 to go to state 2 and 0.10 to go to sate 4, during one period of time.

4: **Broken**, transition probability of 0.20 to go to state 1 and 0.60 to go to state 2, during one period of time.



$$Q = \begin{pmatrix} -0.95 & 0.95 & 0 & 0 \\ 0.05 & -0.30 & 0.20 & 0.05 \\ 0.10 & 0.40 & -0.60 & 0.10 \\ 0.20 & 0.60 & 0 & -0.80 \end{pmatrix}$$

b)
(1) A+B+C+D=1
(2) 0.05B+0.1C+0.2D=0.95A
(3) 0.95A+0.4C+0.6D=0.3B
(4) 0.2B=0.6C ==>B=3C ==> C=1/3B
(5) 0.05B+0.1C=0.8D ==> C=8D-0.5B

(3)+(4) ==> 0.95A+0.6D=0.5C ==> C=1.9A+1.2D
(3)+(4)+(5) ==> 6.8D=1.9A+0.5B ==> B=13.6D-3.8A
(2) ==> 0.68D-0.19A+0.19A+0.12D+0.2D=0.95A==>D=0.95A ==> C=3.04A ==> B=9.12A

(1) ==> A=0.0709 (7.09 %) B=0.6464 (64.64 %)
C=0.2154 (21.54 %) D=0.0673 (6.73 %)

93.27 % of the time when the bike is whole (states 1, 2 and 3)
Expected time between two entrances in or exits from state *i* is obtained by $1/(\pi_i q_{ii})$. There is only one exit from state 1, so the time between two exits from state 1 is equal to the time between two buys of a new bicycle. The time between two such buys is in average 1/(0.0709*0.95*2) = 7.425 years.

c)

Transitions having an economical effect			
Event	**Time between**	**Amount**	**Amount/year**
Maintenance	1/(0.2154*0.4*2) = 5.8 years	- 500 SEK	- 86.2 SEK/year
Repair	1/(0.0673*0.6*2) = 12.4 years	- 1000 SEK	- 80.6 SEK/year
Buy new from “as good as new”	1/(0.6464*0.05*2) = 15.5 years	- 8000 SEK	- 516.1 SEK/year
Sell bike in “as good as new”	15.5 years	3000 SEK	193.5 SEK/year
Buy new from “worse state”	1/(0.2154*0.10*2) = 23.2 years	- 8000 SEK	-344.8 SEK/year

Sell bike in “worse state”	23.2 years	1000 SEK	43.1 SEK/year
Buy new from "broken"	1/(0.0673*0.2*2) = 37.1 years	- 8000 SEK	-215.6 SEK/year
Sum costs	**-**	**-**	**1243.3 SEK/year**
Sum incomes	**-**	**-**	**236.6 SEK/year**
Total cost	**-**	**-**	**1006.7 SEK/year**

From the total cost of 1243.3 SEK/year, 86.6 % are from buying of a new bike, 6.9 % from maintenance and 6.5 % from reparations. There is also an income, from selling, which makes the total cost for having a bike decrease by slightly more than 19 %, to around SEK 1000 per year.

5.3 Solutions to LCC and RCM

Problem 3.1
In order to compare the alternatives, look at the sum of the costs for each investment - all of the costs discounted to today's value.

For each investment we have:

Cost = Investment + (Preventive maintenance + Cost of failure)*(Sum present value factor) + Rest value*Present value factor

Cost of failure is 20 000*SAIDI+60 000*SAIFI:

	Do nothing	Bury	Double feeding	Switcher
Annual cost of failure	44 900 SEK	19 700 SEK	30 200 SEK	27 800 SEK

The Present value factor and the Sum present value factor are, over a time period of 30 years and with discount rate 10 %:

$$\text{Sum present value factors} = \frac{(1+0.07)^{30}-1}{0.07(1+0.07)^{30}} = 12.4090$$

$$\text{Present value factor} = \frac{1}{(1+0.07)^{30}} = 0.1314$$

The numbers of the switcher investment put into the formula:

200 000 SEK + (4000 SEK +27 800 SEK) * 12.4090 SEK – 40 000 SEK * 0.1314 =
= 589 353 SEK

The same calculations for the other investments give:

Do nothing	Cable	Double feeding	Switcher
557166 SEK	536 576 SEK	594 791 SEK	589 353 SEK

With these assumptions, cable would be the best alternative. When the discount rate is changed, the costs are affected according to the table below:

Discount rate	Do nothing	Cable	Double feeding	Switcher
5 %	690 233 SEK	588 955 SEK	692 640 SEK	679 589 SEK
7 %	557 166 SEK	536 576 SEK	594 791 SEK	589 353 SEK
9 %	461 287 SEK	497 869 SEK	523 721 SEK	523 687 SEK

With a discount rate of 5 or 7 %, the most economical investment alternative is cable. If the discount rate is 9 %, no alternative is profitable.

Problem 3.2

a) RCM can be carried out in different ways. This is only one example of how it could look. First, it should be determined what the structure on component level looks like. Is the whole bike to be seen as a unit, or should e.g. saddle, chain etc. be analyzed separately? Here, the bike is divided into smaller units:
   - Frame
   - Wheel
   - Tires
   - Inner tube / ventilator
   - Pedals
   - Luggage carrier
   - Saddle
   - Chain/Gearwheel
   - Screws, screw nuts and holds
   - Breaking system (Handbrake handle, wire, break pads)
   - Lamps/Reflectors

   Physical borders for each unit should be fixed. In the case with the bicycle and the units above, the fixing of physical borders is fairly straightforward.

   The table on the next page illustrates the split up: unit → function → failure function → failure effect → reason for failure → consequence → maintenance/measure. Before a maintenance measure is decided, the consequence should be investigated. The most serious consequences are personal injuries. Consideration should also be given to the number of times that the failure could occur. An operation method can sometimes be seen as a kind of maintenance. An example is that the driving on glass and curbs should be avoided in order to avoid puncture.

	Function	Failure function	Failure effect	Reason for failure	Consequence	Measure
Frame	Keeping all parts physically together	Does not keep all parts together. This failure is so unlikely that it is not studied further				
	Be clean and nice	Not clean and nice	Wears the bike more than necessary, makes cloths dirty, lost image	The bike has been in dirty places	Costs for inconveniences and washing of cloths, lost image	Inspect the bike after the ride and wash if needed
Wheel	Keeping tires and inner tube physically in place	Does not keep tires and tube in place	Wheel gets askew	Spokes get loose of come off	Cost for adjusting the wheel or buying a new one, uncomfortable to go on a bike with an askew wheel	Tighten spokes once a year, make sure no spokes have come off once a month
	Roll with low friction	Rolls slowly	Heavy to go by bike	Bearings worn or dirty	Bike goes slower	If the bike goes slowly, change or clean bearings
	Be clean and nice	Not clean and nice	Wears the bike more than necessary, makes cloths dirty	The bike has been in dirty places	Costs for inconveniences and washing of cloths, lost image	Inspect the bike after the ride and wash if needed
Tires	Give rise to friction against the ground	Does not give rise to sufficient friction	Slides and the driver can fall	Wear against the ground	Accidents can lead to personal injuries	Inspect the depth of the tire pattern and the quality of the rubber every fall

Inner tube/ ventilator	Shock absorber for the bicycle	Does not work as shock absorber	Flat tire	Puncture, hole in the tube/ ventilator damaged	Costs for alternative vehicles, personal injuries from accidents	Avoid driving over glass or high curbs. Fix tube or change it by failure.
		Does not work well as shock absorber	Uncomfortable to use the bike, damages the rim	Slow leak in the inner tube / ventilator damaged	Damages rim and spokes, uncomfortable to use the bike	If air comes out to fast, control first ventilator and then inner tube
Pedals	Make possible an efficient transmission of work from legs to bicycle	Inefficient transmission of work from legs to bicycle. Failure so unlikely that it is not studied further.				
Luggage carrier	Keeping cargo in place	Does not keep cargo in place	Cargo falls off, or can not be put in place	Tension spring is loose or luggage carrier is askew	Costs and labor for cargo falling off or cargo impossible to put in place	Control before using the bike that the cargo is steadily in place
Saddle	Carry the cyclist and being comfortable	Saddle is not comfortable	Uncomfortable to ride the bike	Wrongly placed, spring system broken, wear	Pain in the backside	By need adjust the saddle or change it.
Chain/ Gearwheel	Low friction transfer of power from pedals to wheels	High friction by power transfer	Heavy to push the pedals	Rusty chain or gearwheel	The biking goes slower	Lubricate the chain once every two months, by need more often
		Does not transfer power from pedals to wheels	Can not cycle	Chain off	Costs for alternative vehicles, personal injuries and chain comes off	Lubricate the chain once every two months, by need more often

Screws, screw nuts and holds	Fasten parts together	Does not fasten parts together	Things come off	Loose screws or screw nuts	Personal injuries if things come off or are loose during the ride.	Tighten screws and screw nuts once per year
Breaking system (handbrake, wire, break pads)	Give rise to enough friction to stop the bicycle	Does not give rise to enough friction	Slow braking	Stretching of wire, wear on break pads	Personal injuries if the breaking effect is to low and leads to an accident	If breaks are starting to feel loose, adjust wire and break pads, or change them.
Lamps (battery driven) / reflectors	Send or reflect light	Does not send or not reflect enough light	Poor visibility in darkness, other drivers cannot see the bicycle	Bad batteries, lamp points in wrong direction, defect light bulb, broken reflectors	Personal injuries, lamps/reflectors do not work, which leads to accidents	If lamp has not been used for long: control before start, change batteries when the light is weak, change light bulb by failure, check reflectors and angle them every 2 months

b)

The calculation should go over 9 years, since that is the longest possible life time. For the LCC-calculation, we calculate the present value of all cash flows and add them together. The costs for the two alternatives are:

Cost = Investment (I) + Maintenance (U) + Price of Embarrassment (S) + Rest value (R)

Calculations in table form can look like this:

Expensive Bicycle

Discount factor	1	0.8	0.64	0.512	0.4096	0.32768	0.262144	0.2097152	0.167772	0.134218
Year	0	1	2	3	4	5	6	7	8	9
Investment (SEK)	6 000									
Maintenance (SEK)		300	330	363	399	439	483	531	585	643
Price of Embarrassment (SEK)		150	180	216	259	311	373	448	537	645
Rest value (SEK)										-300
Sum/year (SEK)	6 000	450	510	579	659	750	856	979	1 122	988
Present value/year (SEK)	6 000	360	326	296	270	246	225	205	188	133
Sum present value (SEK)	8 249									

Cheap Bicycle

Discount factor	1	0.8	0.64	0.512	0.4096	0.32768	0.262144	0.2097152	0.167772	0.134218
Year	0	1	2	3	4	5	6	7	8	9
Investment (SEK)	3 000			3 000			3 000			
Maintenance (SEK)		300	300	300	300	300	300	300	300	300
Price of Embarrassment (SEK)		400	480	576	400	480	576	400	480	576
Rest value (SEK)				-300			-300			-300
Sum/year (SEK)	3 000	700	780	3 576	700	780	3 576	700	780	576
Present value/year (SEK)	3 000	560	499	1 831	287	256	937	147	131	77
Sum present value (SEK)	7 725									

Calculations by hand:
With the same notation as in the problem description, we have for the *expensive* bike:

Cost = **I** + **SPV**(25 %,10 %,9 years,**U**) + **SPV**(25 %,20 %,9 years,**S**) + **PV**(25 %,9 years,**R**) = 8 249 SEK

For the cheap bicycle, the costs for embarrassment, investment and rest value occur every three years. The price of embarrassment increases during three years, and then starts over. We can take this into consideration by computing SPV(25 %,20 %,3 years,S). The result is put as cost at the years 0, 3 and 6. For the *cheap* bicycle, we have now:

Cost = **I** + **SPV**(25 %,0 %,9 years,**U**) + **SPV**(25 %,20 %,3 years,**S**) + **PV**(25 %,3 years,(**I** + **SPV**(25 %,20 %,3 years,**S**) + **R**)) + **PV**(25 %,6 years,(**I** + **SPV**(25 %,20 %,3 years,**S**) + **R**)) + **PV**(25 %,9 years,**R**) = 7 725 SEK

Problem 3.3

Calculate the annual failure cost:
Cost all failures/year = grid length [km] * interruption cost [SEK] * [failure/km/year] * (rate of failures which could be prevented)

	Online	Offline
Cost all failures/year (SEK)	150 000	1 200 000

In table form it could look like this:

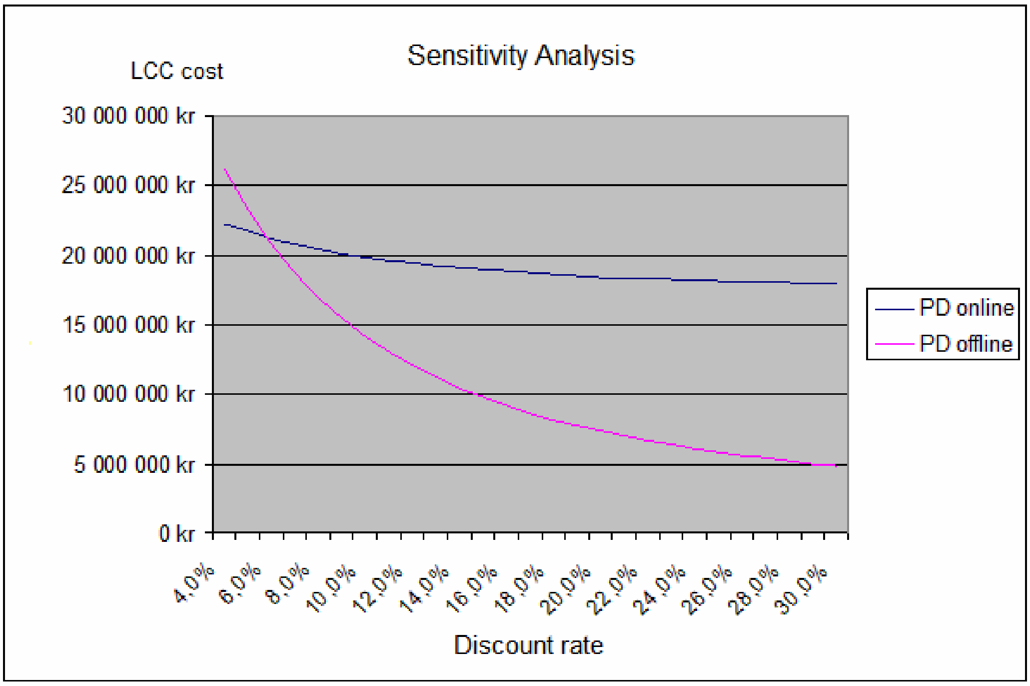
Costs	**Continuous**	**Scheduled controls**	**Discount rate**
Investment [SEK]	17 000 000	0	**7 %**
Monitoring [SEK/year]	100 000	0	
Preventive maintenance of equipment [SEK]	1 000 000	0	
Interval between preventive maintenance	10	0	
Scheduled PD controls offline (including interruption costs) [SEK]	0	1 000 000	
Interval between scheduled PD controls [years]	0	3	
Rest value PD online after 30 years [SEK]	-500 000	0	
Length [km]	100	100	
Interruption cost by failure [SEK]	500 000	500 000	
Failure per kilometer and year, that could be discovered with PD	0,03	0,03	
Rate of failures showing PD, which could be prevented	90 %	20 %	
Cost all failures/year [SEK]	150 000	1 200 000	

Discount rate 7 %

PD online	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2033**	**2034**	**2035**
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**28**	**29**	**30**
Investment [SEK]	17 000 000										
Maintenance [SEK]	0	0	0	0	0	0	0	0	0	0	1 000 000
Monitoring continuous PD [SEK]	0	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000	100 000
Cost of failure/year [SEK]	0	150 000	150 000	150 000	150 000	150 000	150 000	150 000	150 000	150 000	150 000
Rest value [SEK]	0	0	0	0	0	0	0	0	0	0	-500 000
Sum of costs per year [SEK]	17 000 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	250 000	750 000
Present value of costs per year [SEK]	17 000 000	233 645	218 360	204 074	190 724	178 247	166 586	155 687	37 601	35 141	98 525
Sum of present values [SEK]	20 934 712										

PD offline	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2033**	**2034**	**2035**
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**28**	**29**	**30**
Scheduled PD measurement [SEK]	0	0	0	1 000 000	0	0	1 000 000	0	0	0	1 000 000
Cost of failure/year [SEK]	0	1 200 000	1 200 000	1 200 000	1 200 000	1 200 000	1 200 000	1 200 000	1 200 000	1 200 000	1 200 000
Sum of costs per year [SEK]	0	1 200 000	1 200 000	2 200 000	1 200 000	1 200 000	2 200 000	1 200 000	1 200 000	1 200 000	2 200 000
Present value of costs per year [SEK]	0	1 121 495	1 048 126	1 795 855	915 474	855 583	1 465 953	747 300	180 483	168 675	289 008
Sum of present values [SEK]	18 750 702										

In this case, offline-measurements is the best alternative. A sensitivity analysis with respect to the discount rate gives the following result:



With a low discount rate, online-measurement is more economical, while with a high discount rate, offline-measurement is more economical.

Calculation by hand

Present value factor = PV(r,n) $= \frac{1}{(1+r)^n}$ = present value with discount rate r of a cash flow in n years.

Sum of Present values = SPV(r,n)$= \frac{(1+r)^n - 1}{r(1+r)^n}$ = Sum of present values with discount rate r of a cash flow in n years.

In order to use SPV for the preventive maintenance and those offline-measurements that are not made every year, the annual discount rate is changed to a discount rate corresponding to the time period between maintenance actions. If maintenance is done every three years, and until the equipment is 30 years old, the three year discount rate is calculated like this:

$(1+r_1)^3 = 1+r_3$
$r_3 = (1+r_1)^3 - 1$

Then SPV(r_3,10) is used, since there are 10 periods of 3 years in 30 years.
LCC for an investment alternative is:

Cost = Investment + (annual cost of failures + annual operation) * SPV(r_1,30) – rest value * PV(r_1,30) + (cost FU/measurement) * SPV(r_p,30/p)

r_1 is the annual discount rate.

p is the time between the cash flows.

Those costs which come in intervals of several years can also be calculated individually, if the discounting is done each time these costs arise. In that way, the calculation of a discount rate for several years is avoided.

The numbers needed for online PD LCC are:

$$PV(7\ \%, 30) = \frac{1}{(1+0.07)^{30}} = 0.131367$$

$$SPV(7\ \%, 30) = \frac{(1+0.07)^{30}-1}{0.07(1+0.07)^{30}} = 12.40904$$

r_{10} = (1+0.07)^10 – 1 = 0.9671514

$$SPV(96.71514\ \%, 3) = \frac{(1+0.9671514)^{3}-1}{0.9671514(1+0.9671514)^{3}} = 0.8981354$$

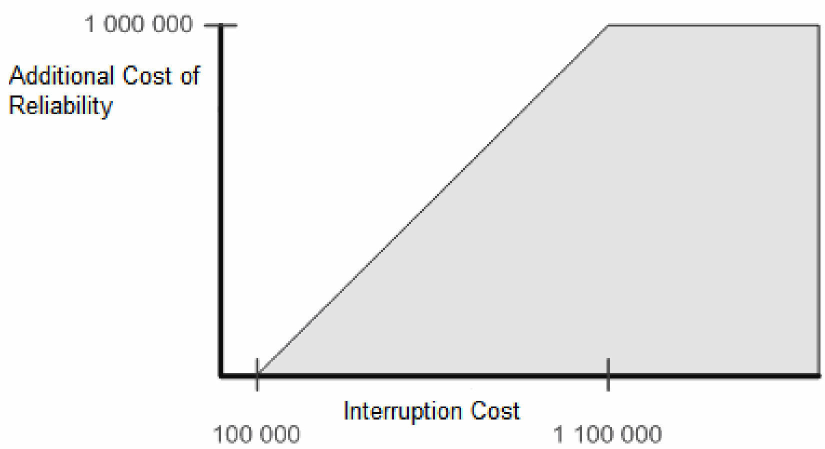
Cost online PD = 17 000 000 SEK + (150 000 SEK + 100 000 SEK) * 12.40904 – 500 000 SEK * 0.131367 + 1 000 000 SEK * 0.08981354 = 20 934 712 SEK

In the same way, the cost for offline-measurement can be calculated. This gives the same result, which is shown in a table.

5.4 Solutions to Network Performance Assessment Model

Problem 4.1
The distribution of electricity, since that part, unlike the selling, is not deregulated. This is because the distribution is a so called natural monopoly.

Problem 4.2
The interruption cost can never be negative. Therefore, the minimal interruption cost is SEK 0, and the maximal SEK 1 000 000 (from the problem formulation). Between these values, the interruption cost is a linear function obtained from:
[reported interruption cost] – [expected interruption cost] (in this case SEK 100 000)



Problem 4.3
Examples of things they have in common are the geographical location of the customers, the annual consumption of the customers and costs for the overlying network. These are usually called objective data. The idea is that these common data should only be circumstances that a potential net owner, who wants to build a network, from the bottom on the same place, cannot affect.

Problem 4.4
That the network is radial means that it lacks every form of reserve feeding (no redundancy). The NPAM takes this into consideration by giving the network owner the right to charge for more components and a longer cable than the radial reference network. The calculation of the expected interruption cost is based on data from redundant networks, which were developed during the evolution phase of the model. All of this is obtained from template functions.

Problem 4.5
When customers belonging to the same transformer are grouped together (the cluster algorithm), there are four limiting conditions stopping the areas from becoming too large. For areas with a high density of customers, two of these conditions are not valid, because the result in these areas is not considered satisfying if these conditions are not taken away. The conditions that are not valid are those concerning the voltage and the current, i.e., how much the voltage is allowed to sink, and the maximal current through a line, respectively. The

immediate surroundings make large low voltage networks possible in densely built-up areas. The definition of immediate surroundings is determined by STEM every year.

Problem 4.6
Monte Carlo-simulation, which is a simulation method. Simulation methods can, for example, work in the following way: random input data (scenarios) are continuously generated to a system during a long fictitious period of time, while the output data of the system are saved. After this, the output data are put together to form average values of the demanded quantities (for example annual interruption cost). With well-balanced assumptions and a simulation over a long period of time, the estimations can be very good, with a low variance.

Analytical methods use mathematical formulae, which, unlike the simulation methods, are not stochastic, on an assumed reliability model. Some simplifications that were made: no simultaneous failures, no consideration to the dimensioning of spear parts, the failure rates are linearly dependent on the length of the line and perfect sectioning.

Problem 4.7
The prioritizing is done only according to the local density of customers; how big the customer is, is not taken into consideration. The single apartment in the city generates the most, followed by the villa, the summer house and the industry.

Problem 4.8
If the debiting grade is higher than 1.00 (in reality, the limit has until now been higher), or if the reduction of reliability cost exceeds maximum, the company risks an inspection by STEM. The debiting grade is calculated as the company's incomes (minus certain costs) divided by the network performance of a possible reduction of reliability cost.