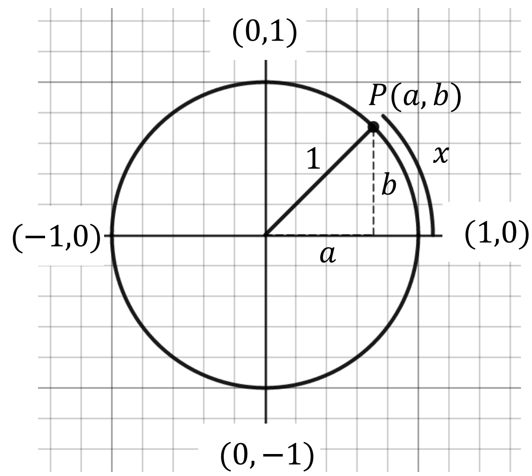
Section 6.4 – Graphing Trigonometric Functions

- In Section 6.1 we looked at the Unit Circle. Remember that the Unit Circle has radius 1.
- Consider the Unit Circle as the Terminal Arm Rotates in a Counter-clockwise direction, with the point (a, b) in the end of the Terminal Arm.

- But from this we get a very interesting Trigonometric Relationship.
- If we consider SOH CAH TOA we get:

$$\mathbf{\sin x} = \frac{Opp}{Hyp} = \frac{b}{1} = \boldsymbol{b}$$

$$\mathbf{\cos x} = \frac{Adj}{Hyp} = \frac{a}{1} = \boldsymbol{a}$$



- So, with that we can make the following observations:

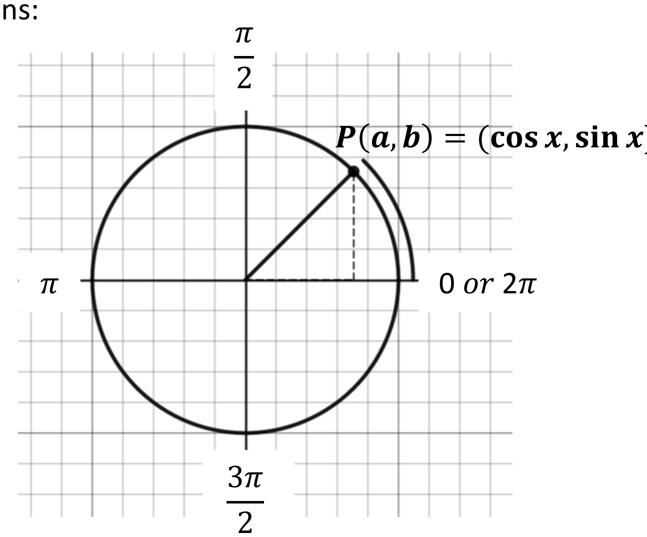
In the Unit Circle, the coordinate at the end of the terminal arm has values where:

The $\boldsymbol{a}$ **coordinate** is the **radian value of** $\mathbf{\cos x}$

The $\boldsymbol{b}$ **coordinate** is the **radian value of** $\mathbf{\sin x}$

Looking at the Unit Circle, you can see that both Cosine and Sine never exceed 1!

Test it, plug $\mathbf{\sin^{-1} 1.2}$ into your **calculator**. You'll get Error 2. Because it Does Not Exist!!



We get the following pattern when we consider how Sine and Cosine vary as x varies.

x	$y = \sin x$	$y = \cos x$
0 *to* $\frac{\pi}{2}$	0 *to* 1	1 *to* 0
$\frac{\pi}{2}$ *to* π	1 *to* 0	0 *to* -1
π *to* $\frac{3\pi}{2}$	0 *to* -1	-1 *to* 0
$\frac{3\pi}{2}$ *to* 2π	-1 *to* 0	0 *to* 1

Graphing a Sine Curve: A Wave Function $\boldsymbol{y = \sin x}$ **for** $\boldsymbol{0 \le x \le 2\pi}$

- Consider the **Radian values in Quadrant 1**, and then **use reference angles** for the other Quadrants.
- Remember to **consider the sign** of the ratio **depending** on the **Quadrant**
- $\boldsymbol{\sin x}$ is: **Positive** in $\boldsymbol{Q1}$ **and** $\boldsymbol{Q2}$
- $\boldsymbol{\sin x}$ is: **Negative** in $\boldsymbol{Q3}$ **and** $\boldsymbol{Q4}$

Quadrant 1					
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	$\mathbf{0}$	$\mathbf{\frac{1}{2}}$	$\frac{1}{\sqrt{2}} = \mathbf{0.71}$	$\frac{\sqrt{3}}{2} = \mathbf{0.87}$	$\mathbf{1}$

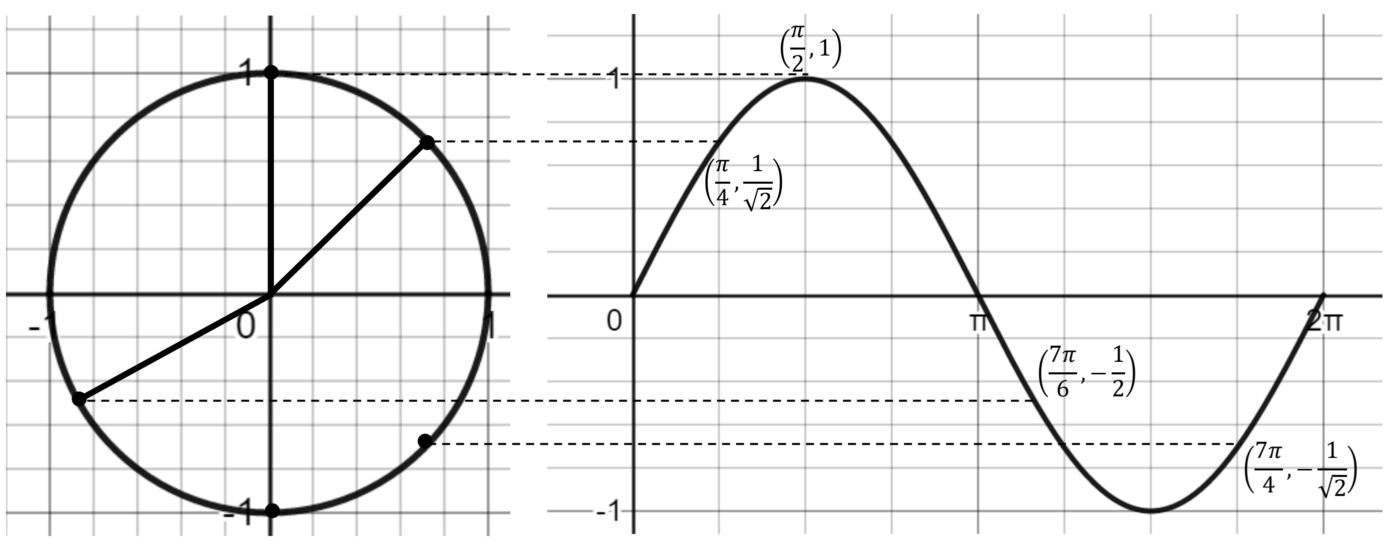
Quadrant 3 (second value is the reference angle)				
x	$\frac{7\pi}{6} = \frac{\pi}{6}$	$\frac{5\pi}{4} = \frac{\pi}{4}$	$\frac{4\pi}{3} = \frac{\pi}{3}$	$\frac{3\pi}{2} = \frac{\pi}{2}$
$\sin x$	$-\mathbf{\frac{1}{2}}$	$-\frac{\mathbf{1}}{\sqrt{\mathbf{2}}} = \mathbf{-0.71}$	$-\frac{\sqrt{\mathbf{3}}}{\mathbf{2}} = \mathbf{-0.87}$	$\mathbf{-1}$

Quadrant 2 (second value is the reference angle)				
x	$\frac{2\pi}{3} = \frac{\pi}{3}$	$\frac{3\pi}{4} = \frac{\pi}{4}$	$\frac{5\pi}{6} = \frac{\pi}{6}$	$\pi = 0$
$\sin x$	$\frac{\sqrt{3}}{2} = \mathbf{0.87}$	$\frac{1}{\sqrt{2}} = \mathbf{0.71}$	$\mathbf{\frac{1}{2}}$	$\mathbf{0}$

Quadrant 4 (second value is the reference angle)				
x	$\frac{5\pi}{3} = \frac{\pi}{3}$	$\frac{7\pi}{4} = \frac{\pi}{4}$	$\frac{11\pi}{6} = \frac{\pi}{6}$	$2\pi = 0$
$\sin x$	$-\frac{\sqrt{3}}{2} = \mathbf{-0.87}$	$-\frac{1}{\sqrt{2}} = \mathbf{-0.71}$	$-\mathbf{\frac{1}{2}}$	$\mathbf{0}$

The curve shows the **height of the terminal arm as it rotates (as the** $\boldsymbol{radian}$ **value** $\boldsymbol{x}$ **moves from** $\boldsymbol{0\ to\ 2\pi}$. You can see that as the Terminal Arm Rotates through the Quadrants, some of the $\boldsymbol{y - axis\ values}$ **are repeated**. $\sin x$, using reference angles, is **Positive in Quadrants 1 and 2 (**$\boldsymbol{0\ to\ \pi}$**)**, then transitions into **Negatives in Quadrant 3 and 4** $(\boldsymbol{\pi\ to\ 2\pi})$**.**

Here is the Terminal Rotating and a Sine Wave Function: $\sin x$. You can see the height of the Terminal Arm during rotation produces the wave as we move along the $x - axis$. This is one full rotation, called a Period.



Graphing a Cosine Curve: A Wave Function $y = \cos x$ **for** $0 \le x \le 2\pi$

- Consider the **Radian values in Quadrant 1**, and then **use reference angles** for the other Quadrants.
- Remember to **consider the sign** of the ratio **depending** on the **Quadrant**
- $\cos x$ is: **Positive** in $Q1$ **and** $Q4$
- $\cos x$ is: **Negative** in $Q2$ **and** $Q3$

Quadrant 1					
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos x$	$\mathbf{1}$	$\frac{\sqrt{3}}{2} = 0.87$	$\frac{1}{\sqrt{2}} = 0.71$	$\frac{1}{2}$	$\mathbf{0}$

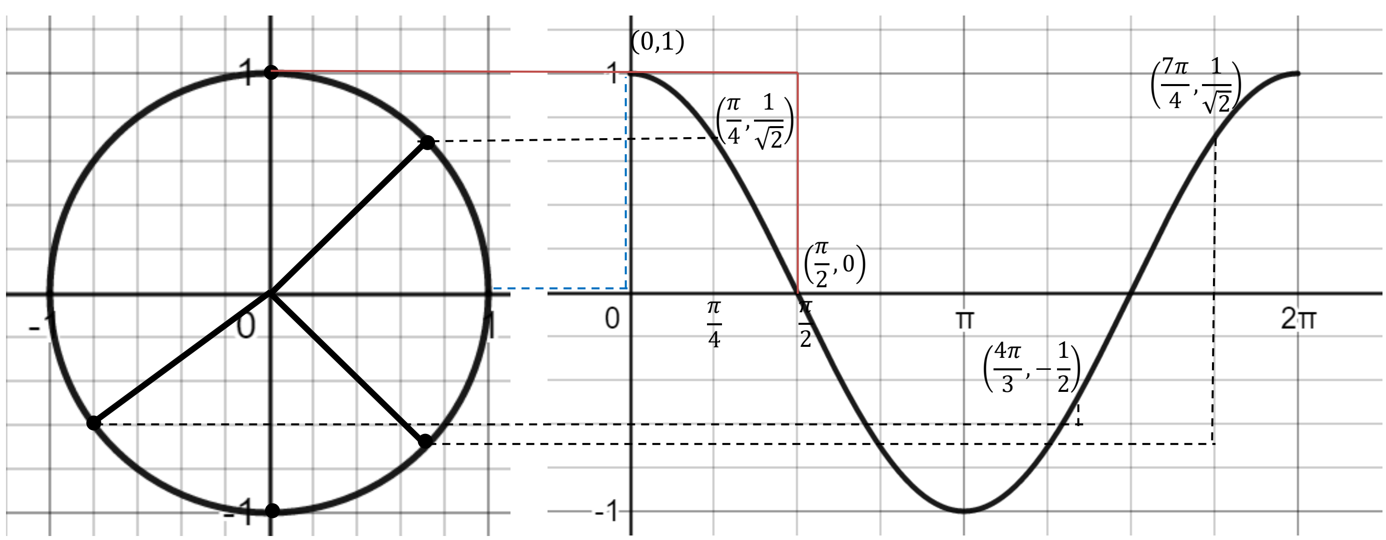
Quadrant 3 (second value is the reference angle)				
x	$\frac{7\pi}{6} = \frac{\pi}{6}$	$\frac{5\pi}{4} = \frac{\pi}{4}$	$\frac{4\pi}{3} = \frac{\pi}{3}$	$\frac{3\pi}{2} = \frac{\pi}{2}$
$\cos x$	$-\frac{\sqrt{3}}{2} = -0.87$	$-\frac{1}{\sqrt{2}} = -0.71$	$-\frac{1}{2}$	0

Quadrant 2 (second value is the reference angle)				
x	$\frac{2\pi}{3} = \frac{\pi}{3}$	$\frac{3\pi}{4} = \frac{\pi}{4}$	$\frac{5\pi}{6} = \frac{\pi}{6}$	$\pi = 0$
$\cos x$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}} = -0.71$	$-\frac{\sqrt{3}}{2} = -0.87$	$\mathbf{-1}$

Quadrant 4 (second value is the reference angle)				
x	$\frac{5\pi}{3} = \frac{\pi}{3}$	$\frac{7\pi}{4} = \frac{\pi}{4}$	$\frac{11\pi}{6} = \frac{\pi}{6}$	$2\pi = 0$
$\cos x$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = 0.71$	$\frac{\sqrt{3}}{2} = 0.87$	$\mathbf{1}$

The curve shows the **horizontal length of the terminal arm as it rotates (as the $radian$ value x move from 0 to 2π**. You can see that as the Terminal Arm Rotates through the Quadrants, some of the $y - axis\ values$ **are repeated**. cos x, using reference angles, is **Positive in Quadrants 1 and 4**, **Negatives in Quadrant 2 and 3.** You may notice it looks similar to a Sine Wave, just shifted.

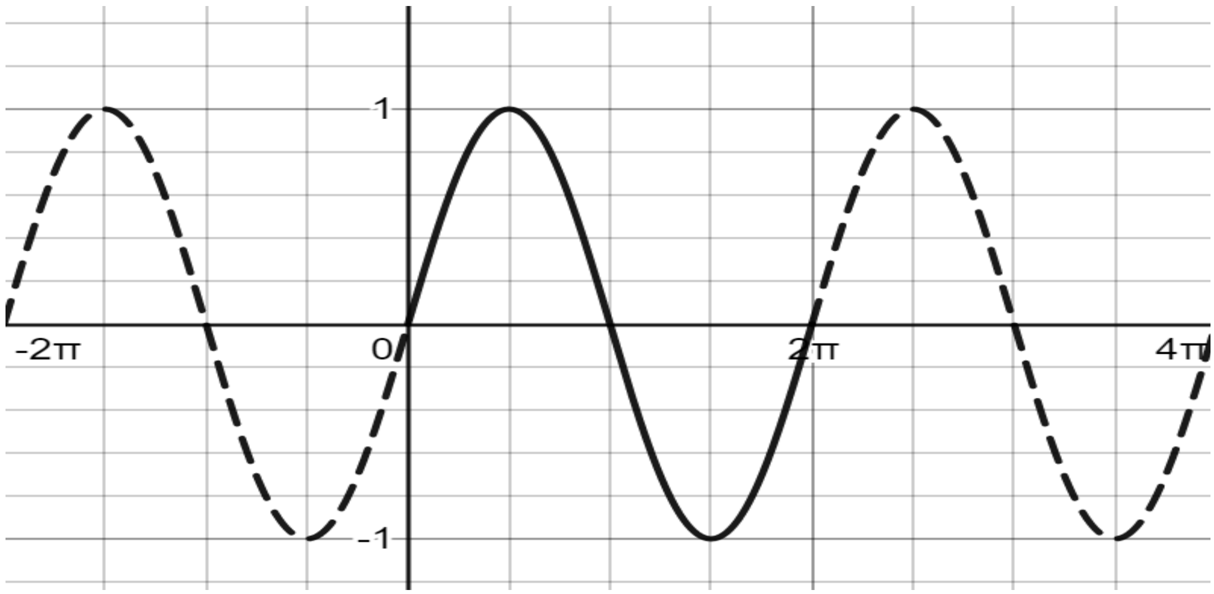
Here is the Terminal Rotating and a Cosine Wave Function: $\cos x$. You can see the height of the Terminal Arm during rotation produces the wave as we move along the $x - axis$. This is **one full rotation**, called a **Period**.



Both **Sine and Cosine Waves extend Horizontally to infinity** in both directions. Each interval, as previously mentioned, runs from $\mathbf{0} \rightarrow \mathbf{2\pi}$ and we call this **A Period**.

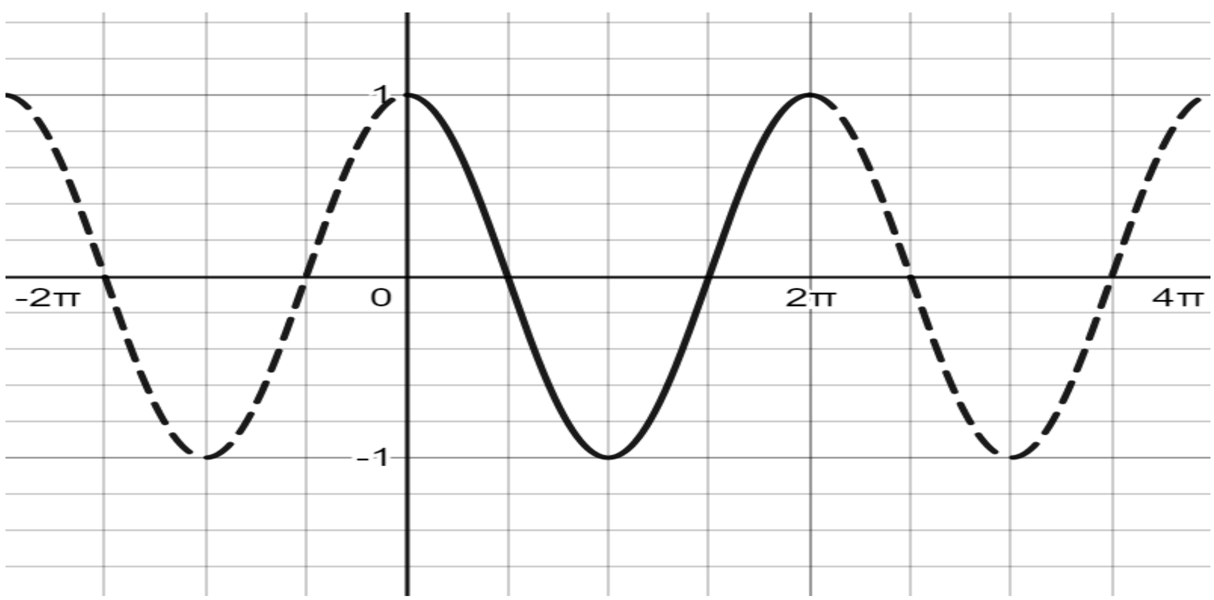
Graph of $y = \sin x$

Period = 2π | Domain: All Real Numbers | Range: $-1 \leq y \leq 1$



Graph of $y = \cos x$

Period = 2π | Domain: All Real Numbers | Range: $-1 \leq y \leq 1$



The above graphs can seem convoluted and challenging to grasp. If it helps, just start with the 4 main angle measures in both $\sin x$ and $\cos x$. Consider the Quadrantal Points $(0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi)$

$\boldsymbol{y = \sin x}$

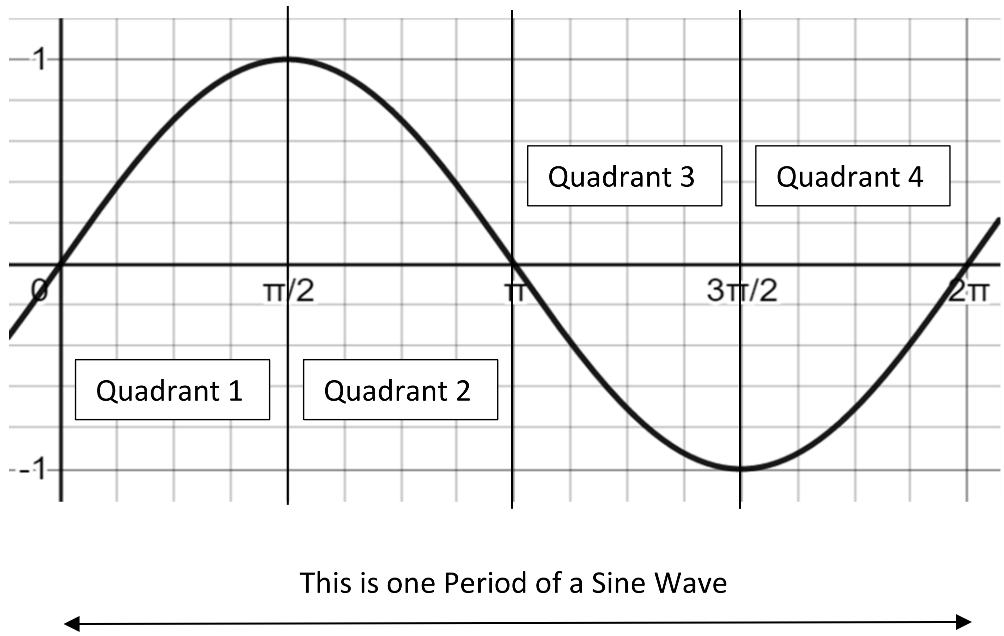
$\sin 0 = 0$

$\sin \frac{\pi}{2} = 1$

$\sin \pi = 0$

$\sin \frac{3\pi}{2} = -1$

$\sin 2\pi = 0$



$\boldsymbol{y = \cos x}$

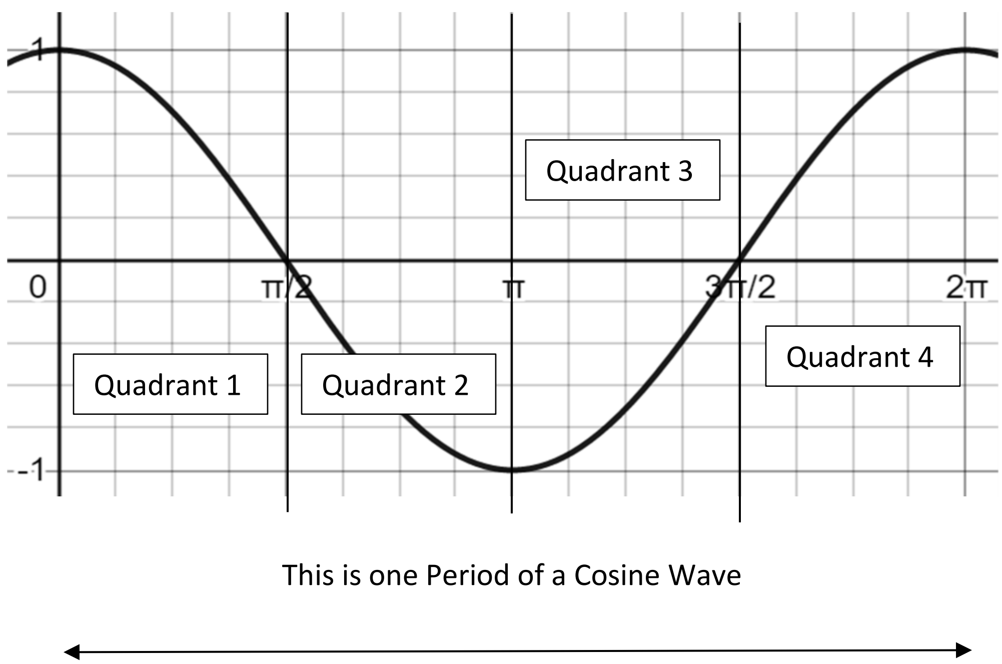
$\cos 0 = 1$

$\cos \frac{\pi}{2} = 0$

$\cos \pi = -1$

$\cos \frac{3\pi}{2} = 0$

$\cos 2\pi = 1$



Amplitude

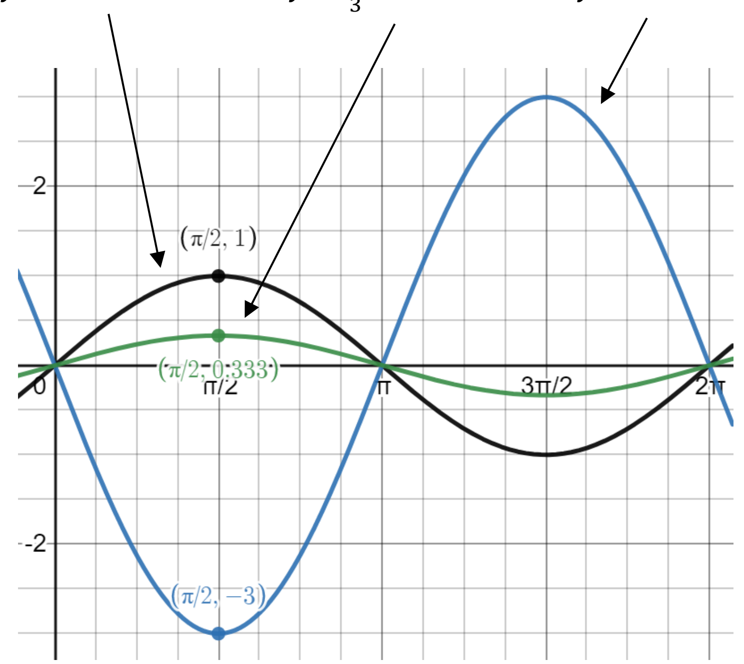
- We when looked at transformation $\boldsymbol{y = af(x)}$ was a **vertical expansion/compression**
- With trig functions we get similar results

$\boldsymbol{y = a\sin x}$ and $\boldsymbol{y = a\cos x}$ means the **height** of our wave is **multiplied** by the **absolute value of $\boldsymbol{a}$ or $|\boldsymbol{a}|$**.

- The **height and depth** of the basic wave always maxes out at **$\mathbf{1}$ and $\mathbf{-1}$ respectively**
- Also, if $a < 0$ (negative), we have a reflection of the $y - values$ in the $x - axis$

The graph below contains the comparisons between:

$y = \sin x$ $\quad$ $y = \frac{1}{3}\sin x$ $\quad$ $y = -3\sin x$



$y = \sin x$; has amplitude of $|1| = 1$

$y = \frac{1}{3}\sin x$; has amplitude of $\left|\frac{1}{3}\right| = \frac{1}{3}$

$y = -3\sin x$; has amplitude of $|-3| = 3$

Period

- We when looked at transformation $\boldsymbol{y = fb(x)}$ was a **horizontal expansion/compression**
- With trig functions we get similar results

For: $\boldsymbol{y = \sin bx}$ and $\boldsymbol{y = \cos bx}$

Consider the period of both is: $0 \le x \le 2\pi$ so that means, for $\sin bx$ and $\cos bx$ the Period is:

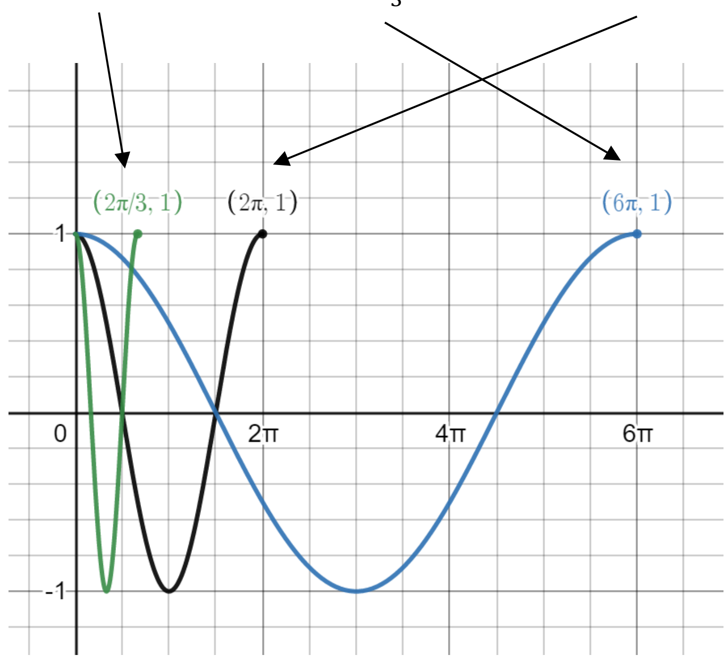
$$0 \le bx \le 2\pi \quad \rightarrow \quad \frac{0}{b} \le \frac{bx}{b} \le \frac{2\pi}{b} \quad \rightarrow \quad \boldsymbol{0 \le x \le \frac{2\pi}{|b|}}$$

- The Period is always positive, and denotes a compression/expansion of the given wave
- To determine the Period of an Expanded or Compressed graph, we use the Formula:

$$Period = \frac{2\pi}{|b|}$$

The graph below contains the comparisons between:

$y = \cos 3x$ $y = \cos\frac{x}{3}$ $y = \cos x$ for $0 \leq x \leq 2\pi$



$y = \cos x$; has Period of $\frac{2\pi}{|1|} = 2\pi$

$y = \cos\frac{x}{3}$; has Period of $\frac{2\pi}{\left|\frac{1}{3}\right|} = 6\pi$

$y = \cos 3x$; has Period of $\frac{2\pi}{|3|} = \frac{2\pi}{3}$

Phase Shift

- We when looked at transformation $\boldsymbol{y = f(x - c)}$ was a **horizontal shift left/right**
- With trig functions we get similar results

For: $\boldsymbol{y = \sin b(x - c)}$ and $\boldsymbol{y = \cos b(x - c)}$

Make sure you have factored out any b value first!

Example: Given $y = \sin(2x - \frac{\pi}{2})$ factor out the 2 to leave x. $y = \sin 2(x - \frac{\pi}{4})$

By doing this we end up with:

Period of: $\boldsymbol{\frac{2\pi}{|2|} = \pi}$

Phase Shift of: $\boldsymbol{\frac{\pi}{4}}$ **to the right**

For the sake of the examples, we will look at Sin Graphs, but the process is the same for Cosine.

Compare:

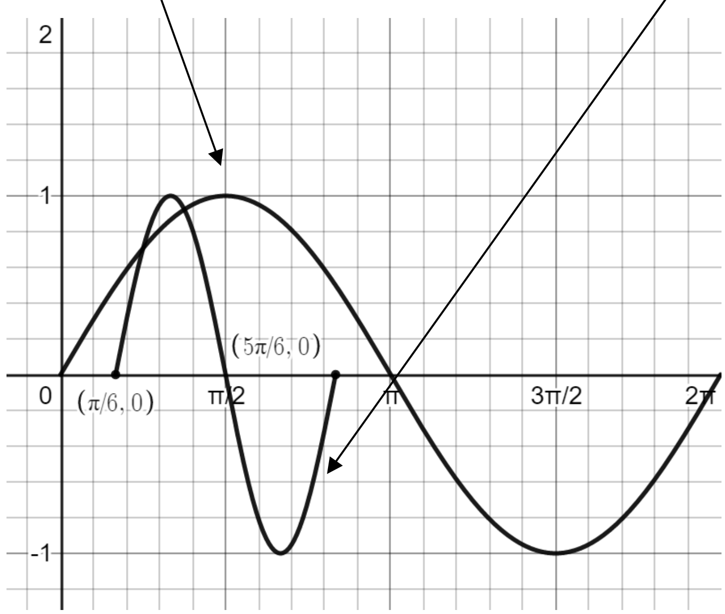
$y = \sin x$ $\qquad$ $y = \sin(3x - \frac{\pi}{2})$ $\qquad$ $y = \sin 3(x - \frac{\pi}{6})$

The graph below contains the comparisons between:

$y = \sin x$ $\qquad$ $y = \sin(3x - \frac{\pi}{2})$ $\qquad$ $y = \sin 3(x - \frac{\pi}{6})$ for $0 \le x \le 2\pi$

$$y = \sin\left(3x - \frac{\pi}{2}\right) \quad \rightarrow \quad y = \sin 3\left(x - \frac{\pi}{6}\right)$$

The second two are actually the same graph!



$y = \sin x$; $\qquad$ has Period of 2π and no Phase Shift

Both

$y = \sin\left(3x - \frac{\pi}{2}\right)$ $\quad$ *and* $\quad$ $y = \sin 3\left(x - \frac{\pi}{6}\right)$

Have:

Period of: $\frac{2\pi}{3}$ and Phase Shift of: $\frac{\pi}{6}$ to the right

So, the graph begins at $\frac{\pi}{6}$

And to find the coordinate that marks the end of the Period, add the phase shift to the Period:

$$\frac{2\pi}{3} + \frac{\pi}{6} \quad \rightarrow \quad \frac{4\pi}{6} + \frac{\pi}{6} = \frac{5\pi}{6}$$

This stuff can get challenging.

Consider:

- The scale of your $x - axis$
- The Period
- Then the Phase Shift
- Faction Basics!!!

Vertical Displacement

- We when looked at transformation $\boldsymbol{y = f(x) + d}$ was a **vertical shift up/down**
- With trig functions we get similar results

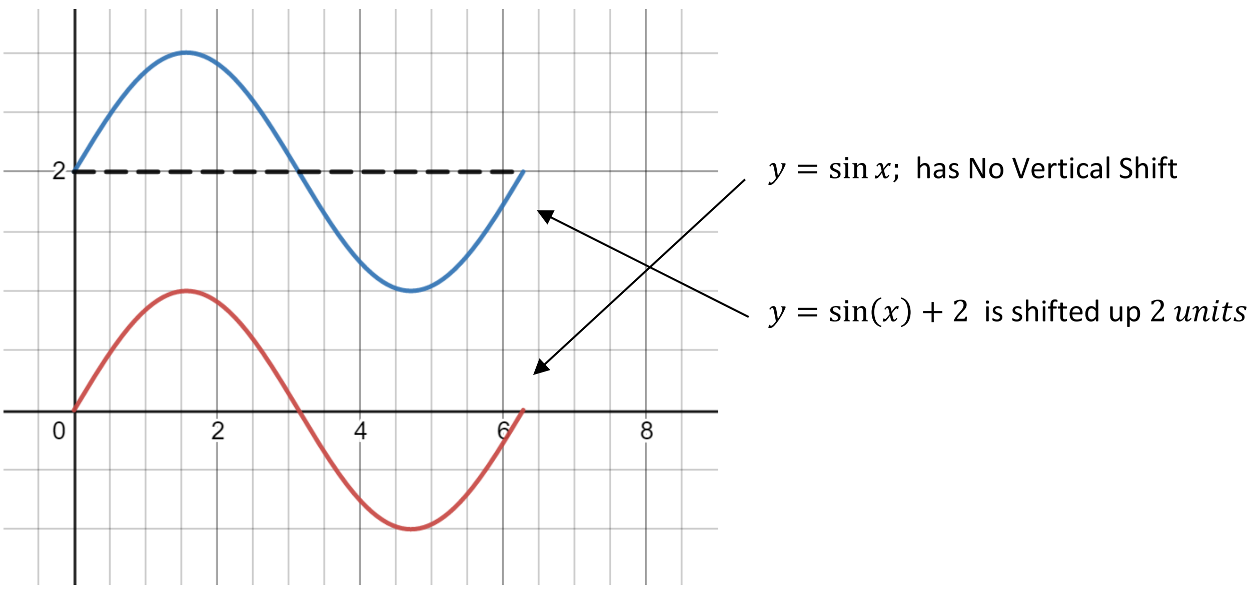
For: $\boldsymbol{y = \sin(x) + d}$ and $\boldsymbol{y = \cos(x) + d}$

If $d > 0$ we have a vertical shift up $d\ units$

If $d < 0$ we have a vertical shift down $d\ units$

The graph below contains the comparisons between:

$y = \sin x$ and $y = \sin(x) + 2$



Summary of Trigonometric Transformations

Consider the form: $f(x) = a \sin b(x - c) + d$ and $f(x) = a \cos b(x - c) + d$

Assume: $a \neq 0,\ b > 0$

Amplitude: $|\boldsymbol{a}|$ **Phase Shift:** $(\boldsymbol{x - c})$ shift **right** $c\ units$ $(\boldsymbol{x + c})$ shifts **left** $c\ units$

Period: $\boldsymbol{\frac{2\pi}{b}}$ **Vertical Displacement:** $\boldsymbol{d\ units}$ $\boldsymbol{d > 0\ up;\ \ d < 0\ down}$

Example 1: Find the amplitude, period, phase shift, and vertical displacement of the following

a) $y = -2\sin\frac{\pi}{6}(x-4)+2$

b) $3\cos\left(\frac{3x}{4}-\frac{\pi}{4}\right)-1$

Solution 1: Do not forget to factor out the $b\ term$, when necessary

a) $y = -2\sin\frac{\pi}{6}(x-4)+2$

Amplitude: $|-2| = \mathbf{2}$

Period: $\frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi}$

$= \mathbf{12}$

Phase Shift:

$(x-4)=0$

$x=4$

Shift $\mathbf{4}$ $\boldsymbol{units}$ to the $\boldsymbol{right}$

Vertical Displacement:

$d=+2$

Shift $\mathbf{2}$ $\boldsymbol{units}$ **up**

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b) $3\cos\left(\frac{3x}{4}-\frac{\pi}{4}\right)-1 \quad \rightarrow \quad 3\cos\frac{3}{4}\left(x-\frac{\pi}{3}\right)-1$

Amplitude: $|3| = \mathbf{3}$

Period: $\frac{2\pi}{\frac{3}{4}} = 2\pi \cdot \frac{4}{3}$

$= \boldsymbol{\frac{8\pi}{3}}$

Phase Shift:

$\left(x-\frac{\pi}{3}\right)=0$

$x=\frac{\pi}{3}$

Shift $\boldsymbol{\frac{\pi}{3}}$ $\boldsymbol{units}$ to the $\boldsymbol{right}$

Vertical Displacement:

$d=-1$

Shift $\mathbf{1}$ $\boldsymbol{unit\ down}$

- Now let's put it all together and graph some trigonometric functions after transformations
- Consider the scale of your $x-axis$ and remember to plot the 4 Quadrantal Points as Guides

Example 2: Graph $y = -2\sin\frac{\pi}{4}(x+3)+1$

Solution 2: Factor if necessary, identify the key information

Amplitude $= \ 2$

Phase Shift = $-3 \ or \ 3 \ units \ left$

Vertical Disp. $= \ 1 \ unit \ up$

Period $= \frac{2\pi}{(\frac{\pi}{4})} = \ \mathbf{8}$

Once you have your Period, divide it by 4 to the distance between the Key Quadrantal Points.

$$\frac{8}{4} = 2$$

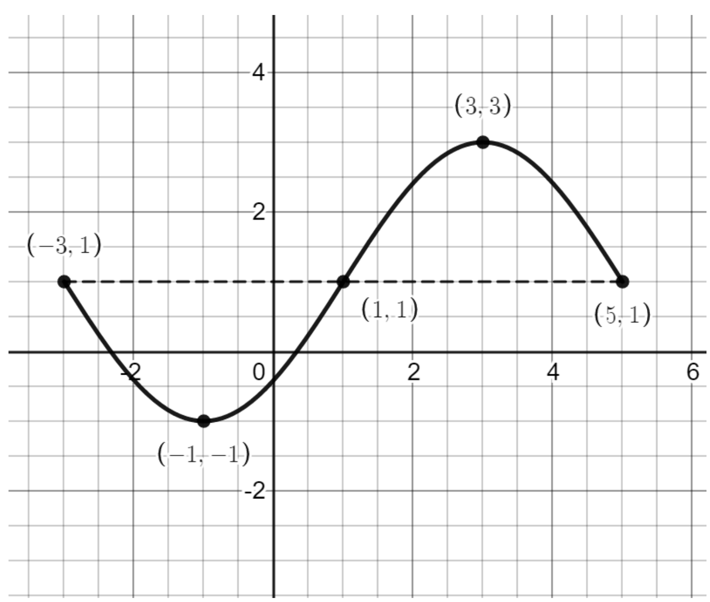
Our **Quadrantal (Peak, Original Height, Valley, Original Height) Points** occur every 2 units.

In this case, being a Sine Wave, we start at -3, but are bumped up 1, $(-3, 1)$, with amplitude of 2.

- Look out! **Then $\boldsymbol{a - value}$ is negative**, so we **start down** instead of up.
- So, the first **valley** occurs **$\mathbf{2}$ units away** but at an **amplitude of $\mathbf{2}$**, so $(\mathbf{-1}, \mathbf{-1})$**.**
- Then we are back to our **starting height**, **$\mathbf{2}$ more units away**, so $(\mathbf{1}, \mathbf{1})$
- Then we hit our **peak $\mathbf{2}$ units after** that, so $(\mathbf{3}, \mathbf{3})$
- Then we are **back to our starting point $\mathbf{2}$ units further** $(\mathbf{5}, \mathbf{1})$

Plot those key points and draw a smooth curve between them.

You'll notice since the Period was a whole number; 8. The scale of the $x - axis$ is 1. This makes for easier plotting and graphing of the curve.



Example 3: Graph $y = 3\cos(2x - 3\pi) - 3$

Solution 3: Factor if necessary, identify the key information

Amplitude $= 3$

Phase Shift = $y = 3\cos(2x - 3\pi) - 3 \quad \rightarrow \quad y = 3\cos 2\left(x - \frac{3\pi}{2}\right) - 3$; $\frac{3\pi}{2}$ units to the right

Vertical Disp. $= 3\ units\ down$

Period $= \frac{2\pi}{2} = \boldsymbol{\pi}$

Once you have your Period, divide it by 4 to the distance between the Key Quadrantal Points. $\frac{\pi}{4}$

Our **Quadrantal (Peak, Original Height, Valley, Original Height) Points** occur every $\frac{\pi}{4}$ units.

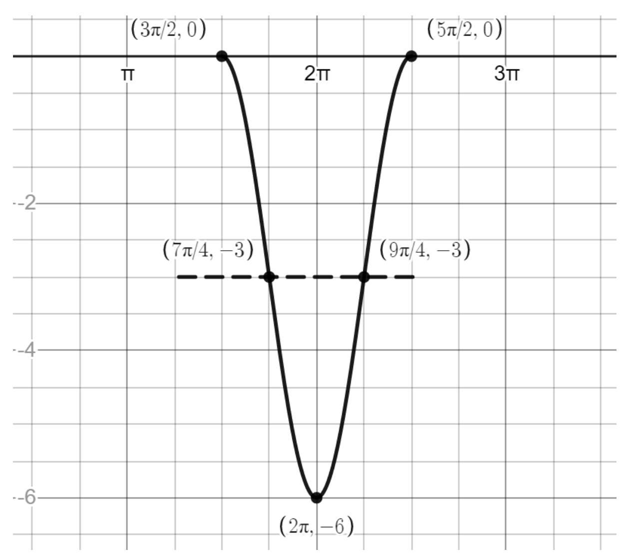
In this case, being a Cosine Wave, we start at the peak, so with an amplitude of 3, and vertical displacement of -3, we stretch from $1\ to\ 3$, then shift down $3\ to\ 0$, and right $\frac{3\pi}{2}$ to $(\frac{3\pi}{2}, 0)$, with

- We **start down** from $(\frac{3\pi}{2}, 0)$
- So, if we move $\pi/4$ units right we end up at $7\pi/4$ but down 3; $(\boldsymbol{\frac{7\pi}{4}, -3})$**,** this is our **midline**
- Then we hit the **valley $\boldsymbol{\pi/4}$ units right** at an **amplitude of $\mathbf{3}$**, so $(\boldsymbol{2\pi, -6})$**.**
- Then we are back to our **midline**, **$\boldsymbol{\pi/4}$ more units away**, so $(\boldsymbol{\frac{9\pi}{4}, -3})$
- Then we return to our **peak $\boldsymbol{\pi/4}$ more units away**, so $\left(\boldsymbol{\frac{10\pi}{4}, 0}\right)\ \boldsymbol{or}\ (\boldsymbol{\frac{5\pi}{2}, 0})$

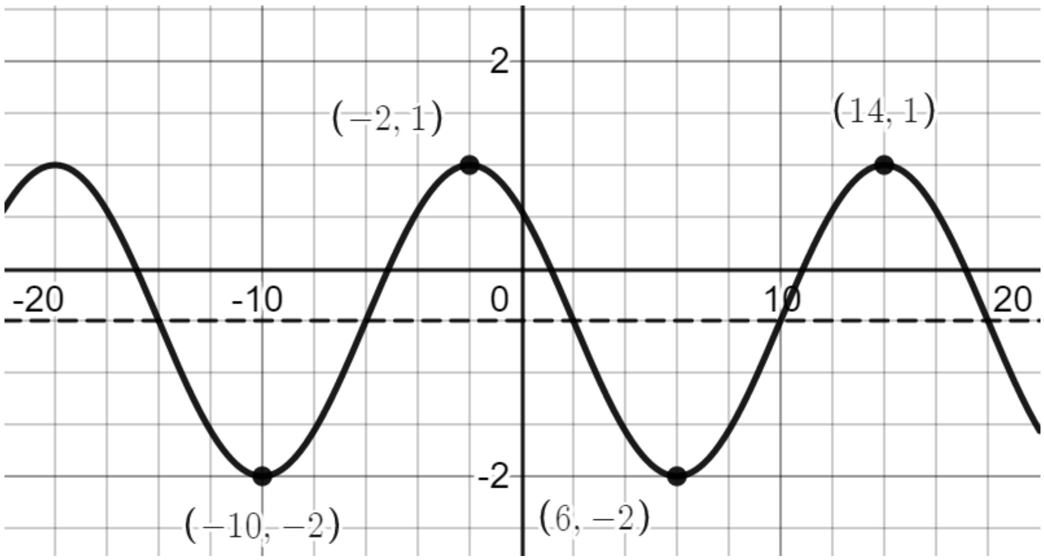
Plot those key points and draw a smooth curve between them.

You'll notice since the Period was π and the phase shift $\frac{\pi}{4}$, we used a scale of $\frac{\pi}{4}$ for the $x - axis$. This makes for easier plotting and graphing of the curve.

We take for granted your ability to find equivalent fractions, the detail is not provided here but assumed. Be careful.



Example 4: Write the equation of the following graph in terms of both Sine and Cosine



Solution 4: You can always find a Sine and Cosine representation, it just depends where you start looking. For a Sine Wave you start at 0, for a Cosine Wave you start at 1 (Or where necessary depending on Vertical Displacement and Amplitude). Considering the infinite flow of a wave, you can start anywhere, so there are infinite possible answers. Watch the scale of the Grid.

Start by identifying the key pieces.

Amplitude: $\left|\frac{3}{2}\right| = \mathbf{\frac{3}{2}}$

Period: $\frac{2\pi}{b} = 16$

$$b = \frac{2\pi}{16} = \boldsymbol{\frac{\pi}{8}}$$

Vertical Displacement:

$$d = -\frac{1}{2}$$

Phase Shift depends on our starting point.

For Sine:

Start at $\boldsymbol{x = -6}$

$$y = \frac{3}{2}\sin\frac{\pi}{8}(x+6) - \frac{1}{2}$$

Start at $\boldsymbol{x = 2}$

$$y = -\frac{3}{2}\sin\frac{\pi}{8}(x-2) - \frac{1}{2}$$

For Cosine:

Start at $\boldsymbol{x = -2}$

$$y = \frac{3}{2}\cos\frac{\pi}{8}(x+2) - \frac{1}{2}$$

Start at $\boldsymbol{x = -10}$

$$y = -\frac{3}{2}\cos\frac{\pi}{8}(x+10) - \frac{1}{2}$$

Graphing $y = \tan x$

We have a specific trigonometric identity to consider when we discuss Tangent.

Recall that:

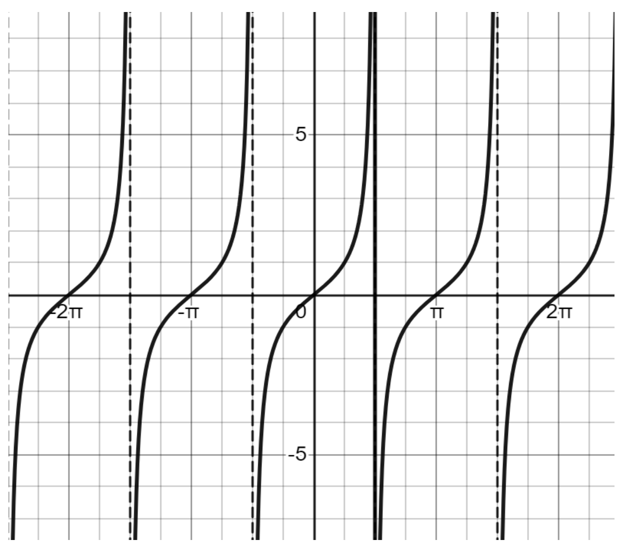
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

This provides us with an issue. We have a discontinuity in the Tangent graph. Why? Because by the fraction, **Tangent is undefined when $\cos\theta = 0$.**

When does this happen? It happens when:

$$\theta = \frac{\pi}{2}$$

And then **every π after that**. Remember our graphing, we have Vertical Asymptotes at this interval.



Period: π

Domain: All Real Numbers, but:

$\frac{\pi}{2} \pm n\pi$, n is an integer

Range: All Real Numbers

Amplitude: None for Tangent

Period of a Tangent Function

Much like Sine and Cosine, the Compression and Expansion of the Period is given by:

$$\boldsymbol{Period} = \frac{\pi}{|b|}$$

Example: Find the Period of: $\tan 2x$

a) $\boldsymbol{Period} = \frac{\pi}{|b|} = \frac{\pi}{|2|} = \frac{\pi}{2}$

Section 6.4 – Practice Problems

1. Which function listed below, matches the details described in the columns

Graph	A	B	C	D	E	F
Amplitude	2	3	2	3	3	2
Period	π	π	3π	3π	$\frac{4\pi}{3}$	$\frac{2\pi}{3}$
Phase Shift	$\frac{\pi}{3}$	$-\frac{\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{3\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$
Vertical Disp.	-2	2	-2	3	3	-3

$f(x) = 2\cos\frac{2}{3}\left(x + \frac{2\pi}{3}\right) - 2$ ______

$g(x) = 3\cos\left(\frac{2}{3}x + \frac{\pi}{2}\right) + 3$ ______

$h(x) = -2\sin 2\left(x - \frac{\pi}{3}\right) - 2$ ______

$i(x) = -2\cos\left(3x - \frac{\pi}{2}\right) - 3$ ______

$j(x) = -3\sin 2\left(x + \frac{\pi}{6}\right) + 2$ ______

$k(x) = 3\sin\left(\frac{3}{2}x - \frac{\pi}{2}\right) + 3$ ______

2. Match the $f(x)$ function with the corresponding $g(x)$ function, such that $f(x) = g(x)$ for all x

a) $f(x) = \sin x$

b) $f(x) = -\sin x$

c) $f(x) = \cos x$

d) $f(x) = -\cos x$

A $g(x) = \cos(-x + \pi)$

B $g(x) = -\sin\left(x - \frac{\pi}{2}\right)$

C $g(x) = \cos\left(x - \frac{\pi}{2}\right)$

D $g(x) = \cos\left(x + \frac{\pi}{2}\right)$

Room to write down thoughts and work through ideas.

3. State the Amplitude, Period, Phase Shift and Vertical Displacement for the graph of each given function.

a) $y = \frac{1}{3}\sin\left(2x + \frac{\pi}{3}\right) - 1$

b) $y = -\frac{1}{2}\sin\pi\left(x + \frac{3}{4}\right) + 1$

c) $y = -4\cos\frac{\pi}{3}(x - 1) + 2$

d) $y = -\cos 2\left(\frac{\pi}{6} - x\right)$

e) $y = 3\sin\left(\frac{2\pi}{3} - \pi x\right) - 2$

f) $y = \frac{3}{2}\cos 2\left(x + \frac{\pi}{4}\right)$

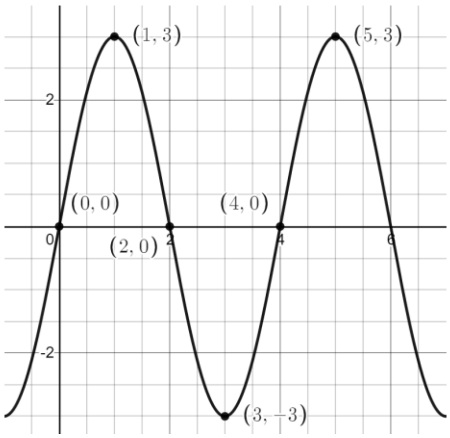
4. What is the Period of the following functions?

a) $y = 2\tan\frac{1}{3}x$

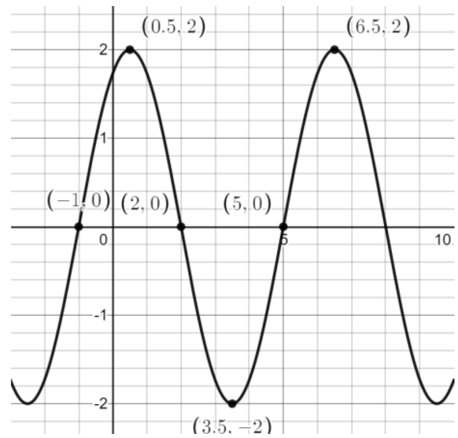
b) $y = -2\tan\frac{\pi}{2}x$

5. Write an equation in the form $y = a\sin b(x - c)$ and $y = a\cos b(x - c)$, where c is the smallest positive number and $a > 0, b > 0$

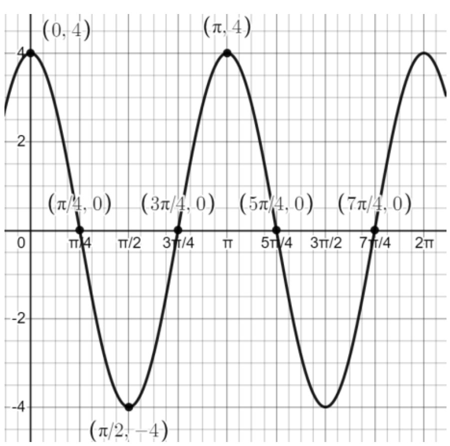
a)



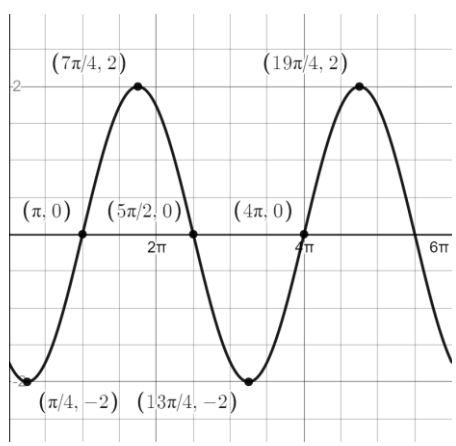
b)



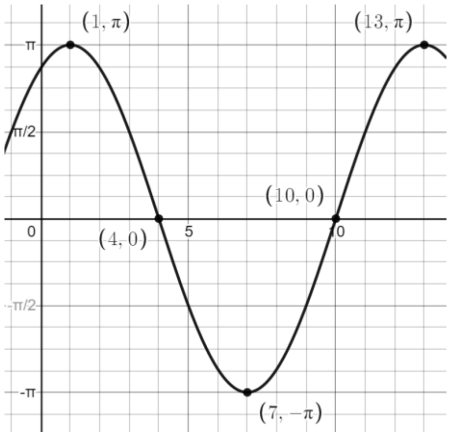
c)



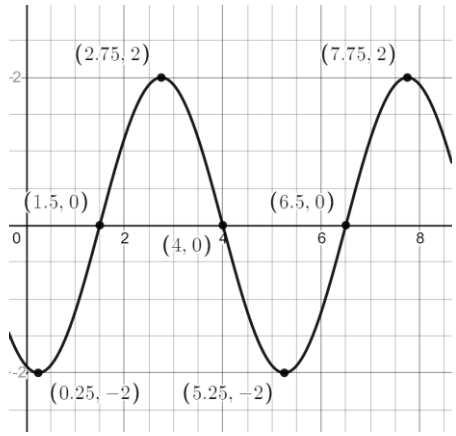
d)



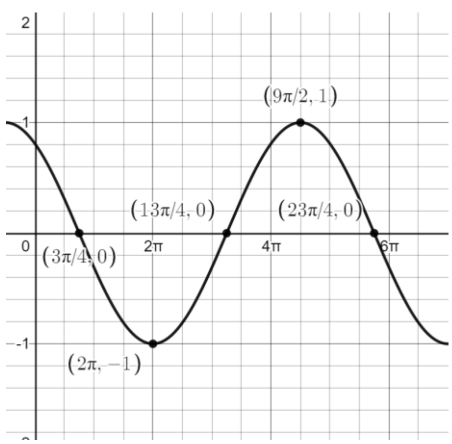
e)


(1, π)
(13, π)
π
π/2
(10, 0)
0
5
10
(4, 0)
-π/2
-π
(7, −π)

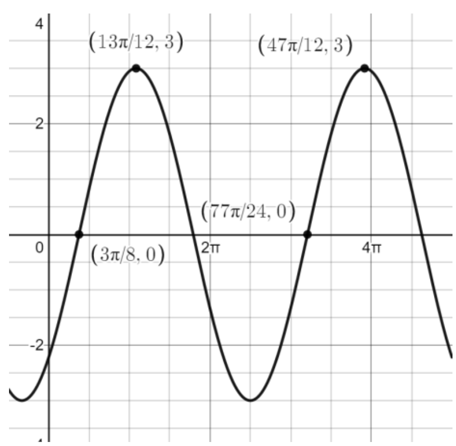
f)


(2.75, 2)
(7.75, 2)
2
(1.5, 0)
(6.5, 0)
0
2
4
6
8
(4, 0)
-2
(0.25, −2)
(5.25, −2)

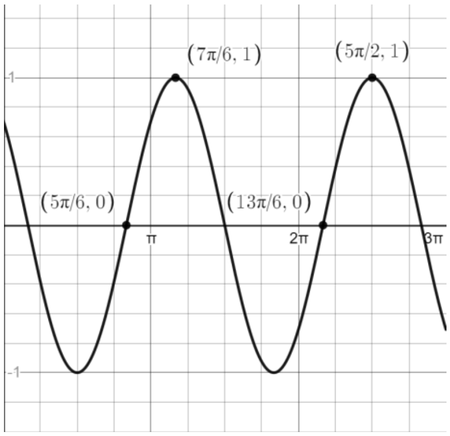
g)


2
(9π/2, 1)
1
(13π/4, 0)
(23π/4, 0)
0
2π
4π
6π
(3π/4, 0)
-1
(2π, −1)

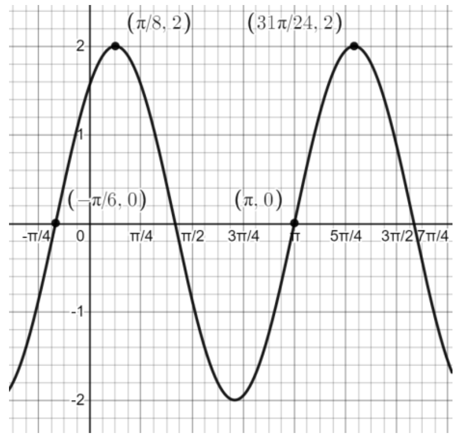
h)


4
(13π/12, 3)
(47π/12, 3)
2
(77π/24, 0)
0
2π
4π
(3π/8, 0)
-2

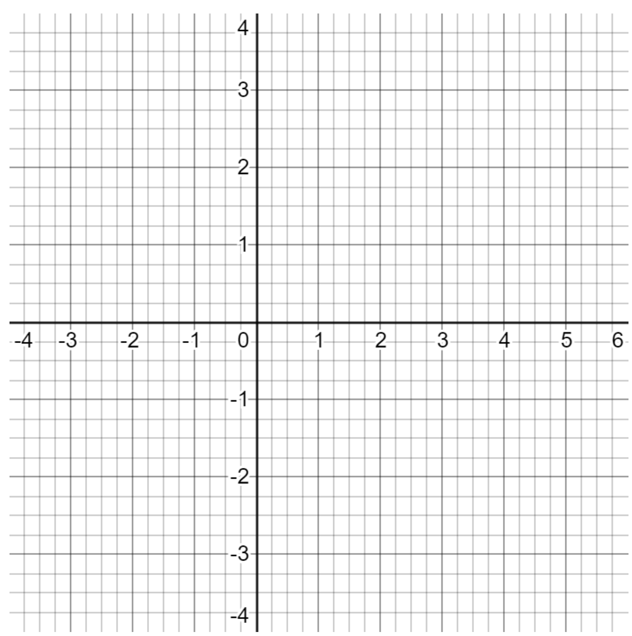
i)



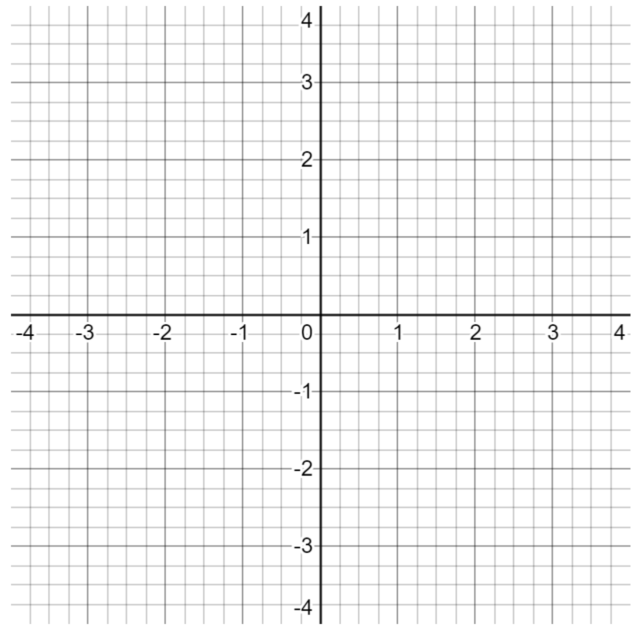
j)



6. Accurately sketch at least one full Period of the graph of: $y = -3\sin\frac{\pi}{3}(x+2)+1$



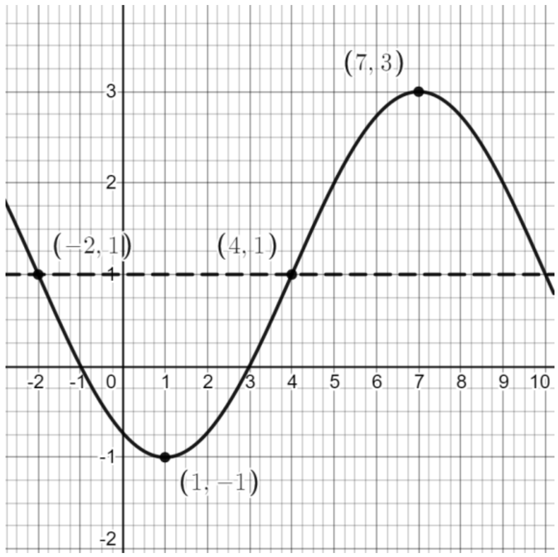
7. Accurately sketch at least one full Period of the graph of: $y = 2\cos\left(\frac{\pi}{2}x + \pi\right) - 1$



8. Find a function in the form $y = a\sin bx + c$ where there is a maximum point at $(2, 3)$ and the next closest minimum point is at $(6, -7)$

9. Find a function in the form $y = a\cos bx + c$ where there is a maximum point at $(2, 3)$ and the next closest minimum point is at $(6, -7)$

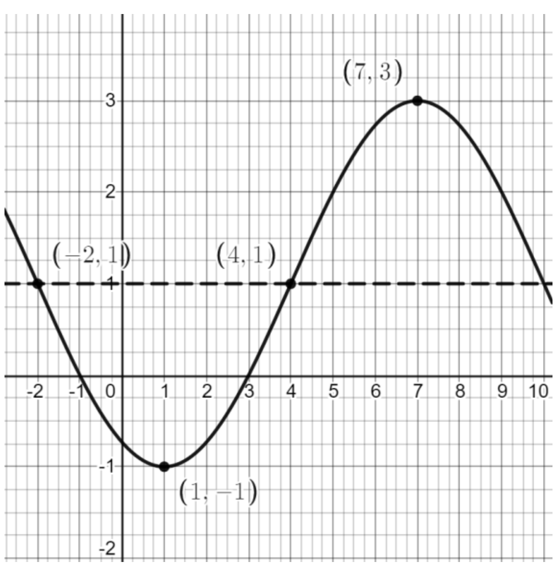
10. a) The graph below describes the function $y = a\sin b(x-c)+d$. Write a sine equation to describe the graph if: **i) $a > 0$ and ii) $a < 0$**



i)

ii)

b) The graph can also be described as a function $y = a\cos b(x-c)+d$. Write a cosine equation to describe the graph if: **i) $a > 0$ and ii) $a < 0$**



iii)

iv)

See Website for Detailed Answer Key

Extra Work Space